Acoustic Wave Powered Climbers

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Abstract

The space elevator tether can be an excellent acoustic conductor for megawatts of low frequency longitudinal vibration power. 1Hz acoustic wave power can be transmitted from the bottom and the top GEO end of the tether, propagating at 28 km/s. Acoustic power is transformed into linear climber thrust and velocity by a 7 kilometer tall (one quarter-wavelength) "acoustic receiver", a pair of motorized wheel-sets coupled by electric cables and a stiff tension tether. Acoustic climbers need only store a few seconds of startup power.

Peak vibration displacement speeds of 100 m/s, and force levels of 40 kN, correspond to acoustic power levels of 2 MW, enough to lift a 2 tonne acoustic climber with a 14 tonne payload at 13 m/s in surface gravity, increasing a few hours later to 100 m/s climb speed at an altitude of 1500 kilometers, where the payload can be transferred upward to an upper-stage acoustic-powered climber. The mathematical analysis is analogized to electronic signal propagation.

Sudden stops from high speed will launch stress waves down the tether, magnified by inverse taper to destructive levels at the ground node attach. Magnetread "wheels" reduce the risk, and may permit higher speed climb.

The launch loop will use dozens of tethers and acoustic climbers to lift more than 2000 tonnes of payload and supplies per hour to launch loop west station, at 50 kilometers altitude.

Nomenclature

$\bar{C}$ distributed capacitance, farads per meter

$\bar{h}$ mechanical:space elevator scale height, km

$\bar{L}$ distributed inductance, henries per meter

$\epsilon(x,t)$ strain (stretch) as a function of distance and time, m/m

$\hat{m}$ tether linear mass density (increase with radius)

λ wavelength, meters

μ standard gravitational parameter, km^3/s^2

ω angular frequency, radians per second

$\psi(x,t)$ displacement (relative shift from unstrained) as a function of distance and time, m

σ Stefan-Boltzmann constant, 5.67e-8 W/m^{2}K^4

A Tether cross section area, m^2

$a(x,t)$ displacement acceleration as a function of distance and time, m/s^2

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c	acoustic power propagation speed, speed of sound
c	electrical: reduced speed of light, m/s
E	Young's modulus, Pascals
f	vibration displacement force, newtons
$f(x,t)$	displacement force as a function of distance and time, newtons
f_a	peak vibration displacement force, newtons
$f_a(r)$	peak acoustic displacement force at radius r, newtons
f_B	acoustic receiver peak back motor wave displacement force
f_D	acoustic receiver peak direct wave displacement force
f_F	acoustic receiver peak front motor wave displacement force
f_I	acoustic receiver peak input wave displacement force
f_O	acoustic receiver peak output wave displacement force
f_R	acoustic receiver peak reflected wave displacement force
G	Gravitational constant, not used here
G	electrical: conductance, A/V
G	vibration displacement conductance, kg/s
$G(r)$	acoustic impedance at radius r, kg/s
k	wavenumber, radians per meter
M	Planetary mass, not used here
P	average acoustic power, watts
P_B	acoustic receiver average back (or reflector) motor power
P_F	acoustic receiver average front (or power) motor power
P_I	acoustic receiver average input wave power
P_O	acoustic receiver average output wave power
P_T	acoustic receiver average extracted power
P_{peak}	peak acoustic power, watts
r	acoustic receiver reflection coefficient
r	radius from earth center, km
T	acoustic receiver extracted power fraction
T_F	Temperature of tether facing flat to the Sun
v	vibration displacement velocity, m/s
$v(x,t)$	displacement velocity as a function of distance and time, m/s
v_a	peak vibration displacement velocity, m/s

$v_a(r)$ peak acoustic displacement velocity at radius r, m/s

v_D acoustic receiver peak direct wave displacement velocity

v_I acoustic receiver peak input wave displacement velocity

v_O acoustic receiver peak output wave displacement velocity

v_R acoustic receiver peak reflected wave displacement velocity

CNT Carbon nanotube

DLR Deutsches Zentrum fr Luft- und Raumfahrt

ESA European Space Agency

GEO Geosynchronous orbit

K Temperature in Kelvins

km kilometer

nm nanometers, 1e-9 meters

nN nanonewtons, 1e-9 newtons

Pa Pascal, one newton per square meter pressure

PV Photovoltaic

SSPS Space solar power satellite

SWR standing wave ratio

UHMWPE Ultra-high-molecular-weight polyethylene

V Volts

Y Yuri, strength/density, 1 m^2/ s^2

1 Introduction

Robert Hooke [3] experimented with the first wire-coupled mechanical telephone in 1665; like a tin can telephone, more than a century before the invention of tin cans. Wire-coupled mechanical telephones were used commercially in the late 1800s, with ranges up to 3 miles [4]. Carbon nanotube tethers, if we ever learn how to crosslink shear force between super-slippery carbon nanotubes, may be far stiffer and less dispersive than the metal wires used for those historic mechanical telephones.

It is surprising, perhaps even embarrassing, that in the half century after Yuri Artsutanov's 1960 proposal for a space elevator, the acoustic properties of a space elevator tether, good and bad, have been ignored. The tether is rigid, strong, and can propagate megawatts of acoustic power, enough to lift a climber at high speed. Unfortunately, it can also magnify shock waves propagating towards the surface, breaking the tether at the ground node.

This paper is a preliminary sketch of the possibilities. Hopefully, it will inspire clever electrical motor engineers to create designs for optimized, robust, high throughput, acoustically-powered space elevator climbers, without heavy photovoltaic panels requiring performance well beyond existing capabilities.

2 Space Elevator Power

"Too many oughta works is a not oughta work"
-Winthrop "Wink" Gross, Tektronix manager, ca. 1985

The space elevator faces many daunting challenges. Challenges should not be multiplied unnecessarily. Powering climbers for their long journey straight up was "solved" by Edwards and Westling in 2002 [1] with a high power laser beam from the ground to a photovoltaic panel on the climber.

There are many problems with this, including laser dangers, clouds, power sources, and focusing. Photovoltaic output voltage (and efficiency) plummets when PV cells get hot; a climber can only dispose of heat with black body radiation, so PV cells will be too large *and* too inefficient. UV lasers to high bandgap PV cells might help, sourced from multiple locations to avoid clouds and spread the beam beyond the climber, but that would be inefficient at the ground end.

Out of the frying pan ...

A proposed fix was the gossamer solar cells allegedly pictured in figure 1. Except ... the photo in figure 1 is a prototype **solar sail**, which produces micronewtons of light pressure thrust and no electricity at all. Solar sails can only deploy in microgravity, or be laid out on a slippery plastic mat on a DLR/ESA hangar floor as shown. In full gravity, even this "small" solar sail would tear apart without a flat smooth floor underneath. A mission-sized solar sail suitable for propelling a small spacecraft (gradually but continuously) would be far larger than any available indoor space. The first mission-scale solar sails will likely be manufactured in orbit. Solar sails are not relevant to space elevator operations.

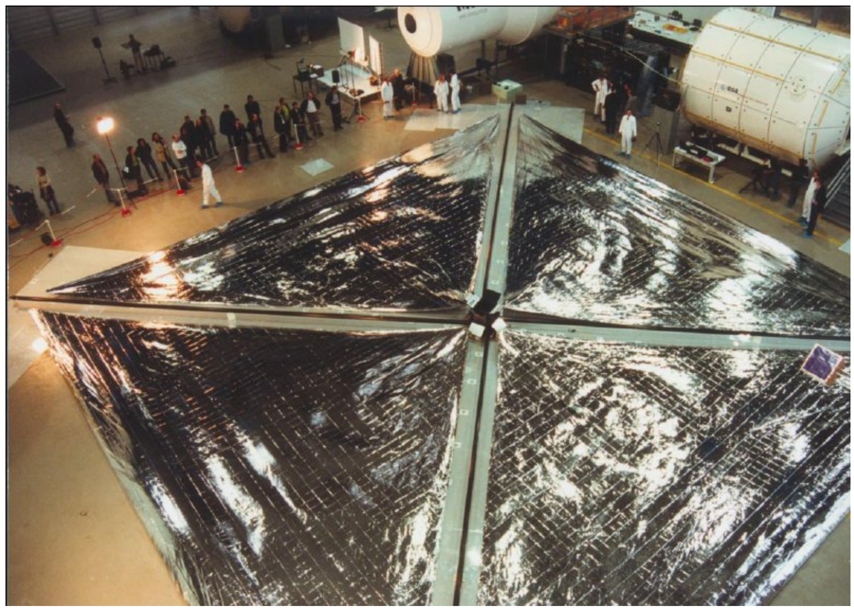

Figure 1: 2003 DLR/ESA 400 m^2 solar **sail** prototype

The 2013 assessment extrapolates from this medium-sized plastic mirror to an enormous 2.9 hectare stack of Sun-powered solar panels, supported (somehow) by many diagonal tethers. Think about force triangles; if the diagonal tethers attach at a 30 degree angle, they put a lateral compression load on the panel equal to half its weight. Like the ESA/DLR sail, the PV film will sag in gravity, and will need an extensive gridwork of support trusses between the main compression spars, capable of supporting the load over the entire angular range between nadir and zenith.

> **Note:** Page 75 of the assessment actually says 29 hectares, 290,000 square meters, for the middle-range numbers. Math error; an array producing 400 W/m^{2}will have an area of 29,000 m^2, not 290,000 m^2.

The hexagonal arrays depicted in the Frank Chase drawings are expected to turn $\pm$ 180 degrees to track the Sun. However, a puppet on strings cannot turn somersaults. The tether is threaded up the middle of the PV panels ... is the climber built around the tether?

These giant arrays, plus the power cables and array pointing motors, must mass less than the 6000 kg total climber mass, while delivering 11.8 MW to the climb motors.

The space solar power community assumes that gossamer space power satellites, orbiting near GEO in microgravity, will mass about 3.5 kg/kw produced, delivering half of that to the terrestrial grid below, after transmission and conversion losses. Space solar power satellite (SSPS) microgravity structures would buckle in a one gee gravity field.

Using the SSPS numbers, 11.8 MW of zero gravity PV array would mass 41 tonnes. Adding structure for full gravity and 360° sun tracking would make the PV array far heavier than that. Super-rigid CNT will certainly help, but it is (theoretically) only 4 times stiffer than existing carbon fiber and 5 times stiffer than Kevlar. Compression members require stiffness, not merely yield strength.

The SSPS community hopes to power a 50 terawatt global economy with 350 million tonnes of space solar power satellites at GEO. That is 25 million climber loads at 14 tonnes per day, or 68,000 years of traffic. Space solar power also has scaling problems.

There have been many recent advances in ultralight photovoltaic arrays for zero-gee deployed arrays. In the lab, quantum dot arrays seem to extract multiple quanta out of one high energy photon (though efficiencies are currently less than 8%).

However, the power from the PV arrays must be gathered into high current cables feeding the climb motors. Carbon nanotube conductors have a theoretical 3x conductivity-per-weight advantage over aluminum, a hard limit set by quantum mechanics and possible only in doped (3,3) armchair tubes. Carbon nanotubes will not be superconductors above cryogenic temperatures, certainly not at the 100C+ temperatures of a PV array cooled by black body radiation.

As power cables get longer, they must get fatter for the same voltage drop and power loss. If they gather more power along their length, they must also get fatter for that, so cable diameter goes up as the square of the panel length, and power cable mass goes up as the cube of the panel length, while panel power production increases only linearly with length.

Only a limited number of cells can be strung in series to increase the voltage. Uninsulated array voltage is limited by destructive arcing to less than 100V; typical satellite PV buses are 28 volts. Experimental insulated arrays have been tested up to 1000 volts, but bubbles, cracks, or sub-millimeter gaps in the coatings are enough to begin an arc that destroys these experimental high voltage panels.

If the space elevator tether is conductive, these huge panels may act as enormous lightning rods, attracting lightning bolts from far away as they climb through the mesosphere. Indeed, the ionosphere is conductive along magnetic field lines, connecting these huge panels to the tops of thunderstorms far to the north or south on the same magnetic longitude.

These effects, and others we haven't imagined yet, cannot be tested on the ground without cubic-kilometer scale vacuum chambers. Until these huge structures are proven at all altitudes on a deployed zero-surface-velocity tether (that is, a complete space elevator tether), they bring enormous technical risk to a space elevator project, even if all the other known problems are solved. What if we build it, and nobody *can* come?

These frightening problems motivated a search for alternatives. Acoustic climbers, using vibration energy on the space elevator tether itself, may be a ground-testable way out.

If acoustic elevators are proven to work, there are many terrestrial applications for the technology; deploying to and from helicopters, or ascending tall towers or steep cliffs, for example. Imagine the outrage that an acoustic tether elevator to the top of Mt. Everest would cause ...

3 Mechanical Behavior Required for a Space Elevator Tether

Space elevator tethers must be extremely strong, but also very stiff and resistant to stretch.

We do not yet know how to make long, ultra-strong tethers from carbon nanotubes.

When we learn how (presumably by replacing zero-shear van der Waals forces between CNT with strong crosslinks that do not weaken the tubes), the material must not suffer from **creep**, the steady accumulation of stretch under cyclic loading. Otherwise, a space elevator will sag over time, like a ribbon of taffy, and fail.

Creep is the gradual stretch of heavily loaded materials. Some strong materials are useful for very brief impact loads, such as armor and bulletproof vests, or storm-resistant mooring ropes. Ultra-high-molecular-weight polyethylene (UHMWPE) materials like Dyneema and Spectra can creep as much as 2% per load cycle [8], and will fail under continuous load. The long molecules do not align completely during manufacturing; they form loops and backwards bends that straighten out under load, but do not contribute to permanent material strength. At the molecular scale, UHMWPE is superstrong, many megayuris, but we have not learned how to line up the molecules.

Two climber transits per day, once up and down, for ten years is 7300 stress cycles; if the 100,000 km elevator tether creeps only 1 part per million per stress cycle, that is 730 km of accumulated creep, shifting tether mass, taper, and strength far below deployment optimum. The tether could become operationally useless before it actually breaks.

Materials such as Kevlar aramid fiber [6] and Toray carbon fiber [7] do not creep when properly manufactured and used safely. However, Kevlar suffers from **hysteresis**; the aramid rings in the molecular chain rotate under tension, and rotate back when released, dissipating energy. This dissipates some of the spring energy in the material. Kevlar fibers are usually woven into cloth and embedded in an epoxy matrix that dissipates more. Composite Aramid fibers are used for structures, such as aircraft fuselages, where hysteresis provides sound dampening and dissipates sudden shock.

Typical damping ratios for Kevlar cloth in epoxy are around 0.8% [11]. This would dissipate a vibration in a Kevlar tether by 50of 2 km would be attenuated 50% in a 172 km distance. Acoustic climbers will use longer wavelengths, perhaps 28 km, but if the damping in the space elevator tether was the same as Kevlar, then 86 wavelengths is 2400 km, 1/15th of the length of a space elevator tether.

The strongest commercially available carbon fiber, Torayca T1100g, is twice as strong as Kevlar per kilogram. The hysteresis is low, and creep is negligible. It is an admirable material, probably very expensive per kilogram, and not nearly strong enough for a space elevator tether.

While individual carbon nanotubes are impressively strong and difficult to fracture, this is because the carbon atoms bind to each other very strongly, and to anything else very weakly, including other carbon nanotubes.

Perfect carbon nanotubes exhibit "superlubricity", which is to shear (side-to-side) friction as superconductivity is to electrical resistance. In 2013, R. Zhang et. al. [10] created atomically perfect centimeter-long double-walled carbon nanotubes, and used an atomic force microscope to pull the inner tube from the outer tube with less than 5 nanonewtons of force. The theoretical force is 1.37 nN for a 2.73 nm outer diameter, 2.39 nm inner diameter double walled tube, which is energy lost at the exit interface, *independent of length.*

Superlubricity is difficult to comprehend; the tiny pull force is the same if the perfect tube pair is a millimeter long, or a km long. Presuming some means of anchoring the ends of the outer tubes at one end, and the inner tubes at the other, a perfect bundle of hexagonally close-packed double wall tubes would have inner to outer pullout strength of 210 MPa and a density of 1950 kg/m^3, a specific pullout strength of 110 kiloYuris, or a support length of 11 km. This is comparable to the 100 kiloYuri specific strength of 6061 aluminum [13]. However, "superlubricity creep" is 100 percent; once DWCNT inner to outer tubes are stretched, they stay stretched.

Superlubricity means there is no way to grip the ends of this perfect bundle; the layer of tubes just inside the outer layer of tubes will slide right out. So, the actual strength of this perfect bundle is only the skin around the outside; the strength may be less than the surface tension of water.

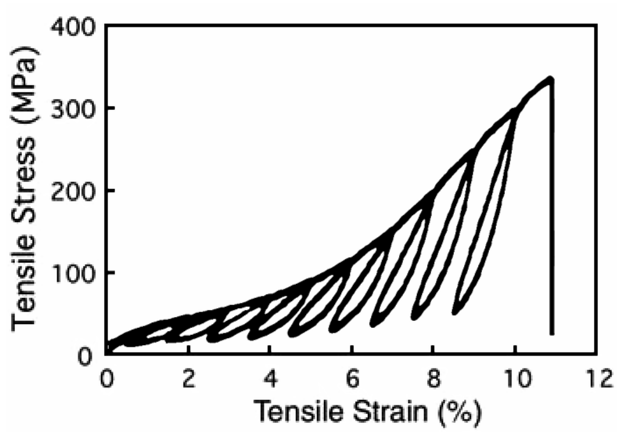

Figure 2: Weakness, Hysteresis, Creep, and Failure for a CNT Yarn

Fortunately for rope makers, the CNT graphene gridwork is often imperfect; bumps or globs of carbon or other materials replace some of the graphene. Those imperfections can bond to other carbon nanotubes, and redistribute stress between them [16]. Unfortunately, the imperfections destroy most of the molecular-scale strength for which we admire CNT. Practically speaking, randomly distributed imperfections also randomly distribute forces in a yarn composed of carbon nanotubes; axial forces will load only a fraction of the fibers.

CNT yarns made to date have terrible creep and hysteresis. Figure 2 (copied from figure 3b of Zhang [9], 2004) shows the cyclic stress/strain behavior of a carbon nanotube yarn. The loops are hysteresis; the path to stress and the path out of stress are different, and the area of the curve represents energy lost, far worse than Kevlar. The creep, the 1% increase in strain (length) with each loading cycle, is far worse than UHMWPE. After 10 stress cycles, to loadings of 350 MPa (perhaps 200 kYuri), the yarn fails. This isn't as weak as superlubricity; defects in and between the CNT bind them somewhat more strongly than van der Waals force, but not by much; this material is 140 times too weak for a space elevator, and 14 times weaker than T1100g. Even if it was much stronger, the creep behavior would cause unacceptable sagging in a space elevator after every climber passage.

The above is illustrative; much stronger fibers have been made in the lab, best values for microfibers approaching 9.5 MYuri (table 1 in [16], published in 2015, though the best data point is from 2007), but averages are lower, and long tethers fail at their weakest point, not at their average. Further, for these very small fibers, it is difficult to establish their actual density and mass, so the 9.5 MYuri number has a wide range of uncertainty.

Another important material property for the tether is its thermal stability over wide temperature ranges, ranging from 330K to less than 20K.

The temperature is highest if the tether is flat to the Sun. The power absorbed is the albedo times 1366W/m^2, the power emitted (on each side of an approximately flat surface) is the albedo times σT^4, where σ is the Stefan-Boltzmann constant, 5.67e-8 $\text{W/m}^2\text{K}^4$. Thus T^4 = 683 W/m^2 / 5.67e-8 $\text{W/m}^2\text{K}^4$ and T = 330K .

However, the tether can twist and face with any angle towards the sun; if it has a slight curvature, the area facing the Sun is not zero, but reduced. If the tether is perpendicular to the Sun, with 20% curvature, it will cool to 220K.

We can expect a tether to twist with altitude; there are no significant restoring forces keeping it flat, except during the temporary transit of a climber, and even slight differences between center and edge stress will twist it helically. That will create regions of varying temperature over the length of the tether, presumably with varying mechanical characteristics. It may even oscillate, heating and cooling, depending on how temperature affects helicity. The temperature differences may be acoustically dispersive and reduce the power reaching a climber far from the transmitter. The rapid temperature swings of the tether as it twists (perhaps hundreds of times per day) could age it quickly.

During the spring and fall equinoxes, the tether will be in full eclipse for an hour every day, and the tether temperature at GEO will drop to about 35 Kelvin. The only heat source is the 250K night sky of the earth, barely peeking 8.7 degrees over the end of the tether, and appearing as $(r_{earth}/r_{geo})^3/(3\pi) \approx 1/2723$ of the Lambert-law-weighted night sky at GEO. The rest of the sky is 2.7K deep space. $35\text{K} \approx 250\text{K} \times \sqrt[4]{1/2723}$. The apex anchor is much further from the "hot" nighttime sky of the Earth; tether temperatures at the apex anchor will be even lower, perhaps 18K, while the apex anchor itself is massive enough to stay warm. A few times per year, the outer tether may be among the coldest objects in the inner solar system.

This analysis assumes that the binder between nanotubes is strong enough to prevent creep and hysteresis losses. Average atomic scale thermal vibrations of carbon atoms at 330K are 830 m/s, and peak thermal speeds are many times that. This suggests that creep (the stretching and gradual failure of the tether) caused by acoustic tether vibration will be much smaller than creep caused by thermal vibration and high static loading.

For the purposes of this paper, I assume a material solution will be found. It must offer at least the 50 GPa, 38 MYuri strength cited in the 2013 space elevator assessment [17] in spite of frequent thermal and mechanical shocks. In addition, the creep (and related mechanical hysteresis) must be very close to zero, far less than Kevlar, so that the 100,000 km tether does not appreciably stretch after decades of the large daily cyclic stresses of vehicle passages and thermal shocks during eclipse.

There is a known material that already achieves these goals - perfect single crystal diamond, with a theoretical tensile strength of 95 GPa [14]. Diamond is stronger than any testing machine, so we only have lower bounds on strength, not empirical measurements. Diamond is currently very expensive and difficult to synthesize, but 50 GPa CNT yarns may be impossible. Other 3-dimensional carbon allotropes, such as C-centered orthorhombic C8, are even stronger than diamond [12], and may be easier (or harder) to synthesize. Improving our diamond synthesis capabilities, perhaps growing diamond fibers outwards from a CNT "seed", may be the "least impossible" way forward. Diamonds are a tether's best friend.

4 Mechanical to Electrical Analogies

The space elevator tether behaves like a lossy, leaky, impedance-tapered electrical signal transmission line. That similarity can be used to borrow a lot of mathematics. 200 years ago, Georg Ohm began the development of electrical circuit theory by analogizing electricity to better understood fluid and mechanical analysis. In the 20th century, electronic analysis developed rapidly, creating a vast trove of mathematics that mechanical engineers can analogize to.

The straightforward analogies of Ohm converted force to voltage and velocity to current. These analogies seem obvious, and work well for charge and electrostatic forces. However, modern technology uses electrical motors based on magnetics, not macro-scale electrostatics; large scale electrostatic forces are quite small compared to magnetic forces, which is why electrical motors (from your CD player to the Large Hadron Collider) use magnets and magnetic fields, not electrostatics.

Converting between electrical and magnetic domains (especially if magnetic motors are involved!) is easier using the **inverse analogy**, force to current and velocity to voltage. The mathematics are similar, but macroscale electro-mechanical systems (more accurately, "magneto-mechanical systems") using rotating and linear magnetic motors are easier to understand.

The voltage on a rotating magnetic motor is proportional to its rotation rate, the current is proportional to torque, while the rotational power is torque times rotation rate. For a linear magnetic motor (such as the magnetread motor described in section 6) the current is proportional to force, and the voltage is proportional to velocity. The "inverse analogy" will help us unify the analysis.

In the inverse mechanical/electrical analogy [19], meters, watts, and joules transfer directly between domains. The fundamental electrical unit in the MKS / SI system of units is the Ampere (the unit of current); all other electrical units derive from that.

I will use the electrical conductivity description and symbol "G" for the mechanical domain. This may sometimes be called mobility or admittance for the real and complex versions of kg/s, and the same units are used for flow, but these are confusing.

Mechanical				::	**Electrical**			
		units					**units**	
Energy	E	Joules	kg m^2/s^2	::	Energy	E	Joules	J = W s
Power	P	Watts	kg m^2/s^3	::	Power	P	Watts	W = V A
Displacement Force	f	Newtons	N = kg m/s^2	::	Current	I	Amperes	A
Displacement Velocity	v		m/s	::	Voltage	V	Volts	V = W/A
Conductance	G		kg/s	::	Conductance	G	Siemens	S = A/V
Impedance	Z		s/kg	::	Resistance	R	Ohms	Ω = V/A
Momentum	p		kg m/s	::	Charge	Q	Colombs	C = A s
Displacement	ψ		m	::	Flux	Φ	Webers	Wb = J/A
Mass	m	Kilograms	kg	::	Capacitance	C	Farads	F = A s/V
Compliance	1/k	m/N	s^2/kg	::	Inductance	L	Henries	H = V s/A
Stiffness	k	N/m	kg/s^2					

Table 1: **(inverse) electrical to mechanical analogy** between tethers and electrical signal cables. Energy, power, distance, and time are the same in the mechanical and electrical domains. Mechanical stress force becomes current, mechanical displacement velocity becomes voltage, and the rest of the units follow.

The most important relations for acoustic tether analysis are vibration displacement force f, the vibration displacement velocity v, and the vibration displacement conductance G. V and F refer to climber velocity and gravitational force. We will ignore the effects of fast climb rates on power delivery rates and effective wavelength, which adds a "doppler shift" adjustment to the equations.

For a sinusoidal vibration, the displacement velocity v is related to displacment force f by $v = Gf$. Unidirectional waves travel at a "speed of sound" of $c = \sqrt{Modulus/density}$, perhaps 28 km/s for CNT tethers.

The conductance G for a section of tether is the linear mass density $\hat{m}$ (kg/m) times c, or $G = \hat{m}c$. G is 382 kg/s at the surface node and 2290 kg/s at GEO node, gently tapering between those positions.

The peak unidirectional power propagated down the tether is $P_{peak} = v_a f_a$, and the average power P is half of that, $P = \frac{1}{2} v_a f_a$. Since $v_a = G f_a$, the average power is also $P = \frac{1}{2} {f_a}^2 / G = \frac{1}{2} G v_a^2$. Superstrength carbon nanotube tethers will not only have high acoustic velocities, they will also support high displacement forces at moderate displacement velocities. This allows a thin tether to carry megawatts of power.

We are not working at the micrometer scale (where electrostatics dominate), so we should ignore the old electrical charged fluid analogies and think about magnetic/electric circuits instead. Remember that the above analogy is an aid for translating mathematical analysis between electrical and mechanical domains, not a substitute for understanding what the mathematics describe. Sloppy analogies are the enemies of accurate thought.

> **Note:** The symbol G can also be used for the universal gravitational constant. That G is not directly relevant to space elevators, although the standard gravitational parameter $\mu = GM$, G multiplied by large body mass M, is central to space engineering. G and M are difficult to disentangle and measure separately, and we need not to consider them separately in order to use a large body's measurable μ to compute gravitational forces and orbits to ten decimal place accuracy.

We will ignore the large scale gravity and centrifugal accelerations on the whole tether; for an excellent discussion see [20]. The mechanical equations describing the tether at kilometer scales follow:

The **strain** is $\epsilon(x,t)$, a function of distance x and time t with units of meter/meter, and the linear derivative of displacement $\psi(x,t)$:

$$\epsilon(x,t) \;=\; \frac{\partial \psi(x,t)}{\partial x} \tag{1}$$

The **displacement force** is $f(x,t)$ (newtons N), is:

$$f(x,t) \;=\; EA\frac{\partial \epsilon(x,t)}{\partial x} \;=\; EA\frac{\partial^2 \psi(x,t)}{\partial x^2} \tag{2}$$

... where EA is the tether linear spring constant in newtons, and also the tether's Young's modulus E times the cross section A.

Velocity $v(x,t)$ is the time derivative of displacement. **Acceleration** $a(x,t)$ is the time derivative of velocity, and the second time derivative of displacement. Acceleration is also the displacement force $f(x,t)$ divided by the linear mass density $\bar{m}$ (in kg/m):

$$v(x,t) \;=\; \frac{\partial \psi(x,t)}{\partial t} \;(3.1) \qquad a(x,t) \;=\; \frac{\partial^2 \psi(x,t)}{\partial t^2} \;=\; \frac{f(x,t)}{\bar{m}} \;=\; \frac{EA}{\bar{m}}\,\frac{\partial^2 \psi(x,t)}{\partial x^2} \;(3.2) \tag{3}$$

The **acoustic velocity** c (in meters per second) is defined by $c^2 = EA/\bar{m}$. Combining the above, the displacement, tension, strain, and velocity are all described by the wave equation:

$$\frac{\partial^2 \star(x,t)}{\partial t^2} = c^2\,\frac{\partial^2 \star(x,t)}{\partial x^2} \tag{4}$$

... where $\star$ can be one of:

$\star$	variable	ψ	displacement	v	displacement velocity
f	displacement force	a	displacement acceleration	ϵ	strain
	electrical:	V	voltage	I	current

The wave equation describes the propagation of arbitrary waveforms; for acoustic tether power, we will use sine waves of the form $\sin(\omega t + kx)$, where $\omega \;=\; 2\pi\ freq$ is the angular frequency in radians per second.

$k = 2\pi/\lambda$ is the wavenumber in radians per meter, and λ is the wavelength in meters. Given the acoustic propagation speed , $c = \lambda$ frequency $= \omega \;/\; k$.

Note that k is also used for the spring constant, Boltzmann's constant, and thermal conductivity.

Electrical cables use $\bar{L}$ and $\bar{C}$ for distributed inductance and capacitance, similar to distributed compliance $1/EA$ and distributed mass $\bar{m}$. That means we can directly borrow the math from electronic transmission lines. In particular, we can borrow equations from pages 368 to 370 of Potter and Fich [5] for the exponential line.

The propagation velocity of an electronic cable is $c^2 = \bar{L}\bar{C}$, and the conductance is $G = \bar{C}/\bar{L}$. In an electrical exponential line, $\bar{L}$ increases and $\bar{C}$ decreases exponentially:

$$\bar{L} = \bar{L}_0 \; \exp(\kappa x) \quad \bar{C} = \bar{C}_0 \; \exp(-\kappa x) \tag{5}$$

After two pages of math, we learn that if κ is small, that is, the exponential taper is gradual compared to the wavelength λ of the vibration, the line does not attenuate, but propagates power with increased wave voltage and decreased wave current.

We can translate their analysis into more convenient wavelength-oriented units for acoustic vibration, and a maximum cutoff wavelength.

$$EA = EA_0 \exp(x/\bar{h}) \quad \bar{m} = \bar{m}_0 \; \exp(-x/\bar{h}) \tag{6}$$

... where $\bar{h}$ is the characteristic height the space elevator tether, 2762 km on page 136 of the assessment.

The high frequency cutoff for the exponential line condition in [5] can be transformed into a long wavelength cutoff $\lambda < 4\pi\bar{h}$. The tether will pass wavelengths shorter than 9575 km, or vibration periods shorter than 340 seconds. The taper change over height will reflect the long wavelength components of very slowly accelerating and decelerating climbers, which may lead to tether dynamics problems, but it won't affect the propagation of acoustic power (with much shorter wavelengths) from either terminus to the climber.

The acoustic conductance $G(r)$ is proportional to tether cross section, and increases with radius r. Assume the acoustic power P is constant. How does the peak sinusoidal displacement force $f_a(r)$ and the the peak sinusoidal displacement velocity $v_a(r)$ change with radius?

$$P = \frac{1}{2} f_a(r) v_a(r) = \frac{1}{2} f_a(r)^2/G(r) = G(r) v_a(r)^2 \quad \rightarrow f_a(r) = \sqrt{2PG(r)} \text{ and } v_a(r) = \sqrt{2P/G(r)} \tag{7}$$

The peak force is proportional to $\sqrt{G(r)}$ and the peak velocity is proportional to $\sqrt{1/G(r)}$. The cross section of the study tether increases by a factor of 6 between the ground node and the GEO node, and the acoustic impedance $G(r)$ will increase by a factor of 6 as well, from 382 kg/s at the ground node to 2292 kg/s at the GEO node. That means that a 1.91 MW power wave at the ground node (100 m/s $v_a(gnd)$ and 38.2 kN $f_a(gnd)$) will be transformed into a 1.91 MW power wave at GEO node, with V(GEO) = 40.8 m/s (divided by $\sqrt{6}$) and a $f_a(GEO) = 93.6$ kN (multiplied by $\sqrt{6}$). Not a problem for this six times heftier tether.

Propagating downwards from GEO could be a problem; V(gnd) increases by $\sqrt{6}$ from GEO, and F(gnd) decreases by the same amount. Since the taper is nearly constant above 3 Re, downward propagating forces generated above that radius will be amplified as well.

What could possibly go wrong?

The wave equation also applies to compression waves in water; sometimes with catastrophic effects. The exponential line impedance transform describes the efficient coupling of power and energy between regions with very different conductances. This occurred when the power of an undersea earthquake in the high conductance Pacific Ocean coupled with the low conductance shallow bays of the Tohoku coast of Japan in 2011, transforming an ocean's worth of seismic energy into enormous and fast tsunami waves.
"Wheel lock" is an "acoustic tsunami" in a space elevator tether.

5 Potential Catastrophe: Upbound Wheel Lock

If a failure causes one of the wheels on the wheel-set attached near the payload to lock to the payload AND the tether, and there is no mechanism to break the lock and release the wheel, or disconnect the payload, then the length of tether near the payload will be instantly accelerated to payload speed. If the payload is moving upwards at 100 meters per second, that launches a tension wave up and down the tether proportional to the payload velocity (100 m/s) times the tether acoustic impedance (as much as 2292 kg/s on the upper tether, an additional tension of 229.2 KN on the 62.8 mm^{2}cross section tether, 3.65 GPa.

The wave decays in a few seconds, as more tether is entrained. The payload decelerates very rapidly, as shown in figure3. However, the front of the stress wave may propagate without energy loss at full 3.65 GPa amplitude. The peak amplitude is independent of vehicle weight, merely the velocity difference; if the payload mass is small, the rate of stress decay is faster and the payload acceleration higher. For a 16,000 kg vehicle, the deceleration is 14 m/s, and for a 2,000 kg unladen acoustic climber, it is 114 m/s or 12 gees.

As this tension wave propagates downwards towards the ground, the tether grows narrower and the stress pressure increases, multiplying by $\sqrt{6}$ before it reaches the ground node, to a stress of 8.94 GPa.

If the ground node does not immediately pay out more tether before the wave hits (to match the 245 m/s speed of the incoming wave), the wave reflects. Velocity drops to zero, and the tension reflects and doubles at the ground attach, to 17.9 GPa. The ground attach is already under high stress, so this sudden shock might break the tether.

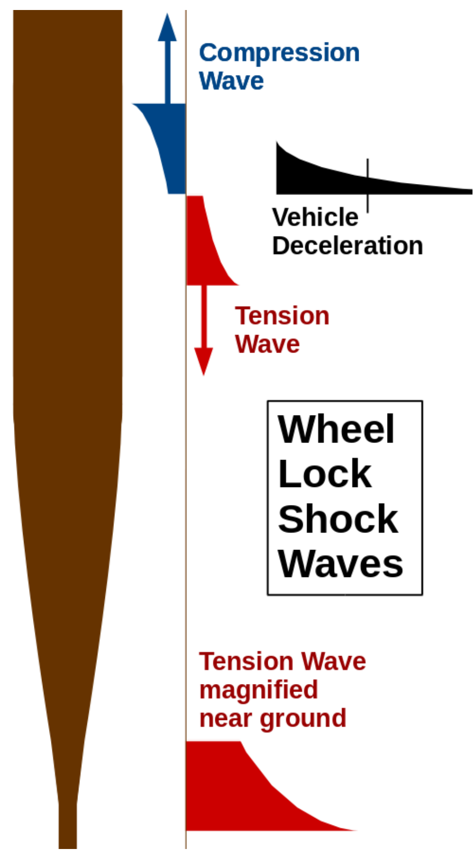

Figure 3: **Wheel shockwaves.** A failed wheel may lock up or fracture, and suddenly stop on the tether. This launches huge stress waves down the tether.

The stress is even higher above a vehicle near the bottom of the tether. Again, the stress wave could reflect and break the tether above a high-inertia drive wheel. With proper planning and agile power control, the climber on the vehicle can release some tension as the wave passes, and harvest much of the stress wave passing through as additional climb power. It will be stressful, bumpy ride for the vehicle, but it might help save the tether.

Again, this phenomena is proportional only to climber speed, not mass, and occurs if a wheel suddenly grinds to a halt on the tether. This might not be a wheel failure; a pinch wheelset encountering large debris embedded in the tether could do the same thing. We presume that low speed large debris will bounce off the tether, and high speed debris will tear a hole, but at velocities somewhere between, tether mesh sized objects will simply plug a hole in the mesh.

A small object accidentally dropped from a climber above the 30,000 km "release to orbit" radius, will remain in orbit until it hits something, possibly the tether itself the next time the object's near-equatorial orbit encounters the tether. The chance of this lineup is small, the object and tether are unlikely to be in the same place at the the same time, but it will be larger than zero. After the object has been perturbed by lunar and solar tides, it will typically have a relative velocity of a few tens of meters per second relative to the tether, which seems like the perfect velocity to stick in the mesh, which has a cross section of 0.1 hectares per kilometer of height, a very large target.

Mitigation 1: The climber might be preceded by a small, expendable mini-climber able to inspect the tether perhaps 100 meters ahead of itself, and "gently" stop before hitting an object. A 20 kilogram inspection robot moving at 100 m/s could also create a wheel lock, but that brief strain wave will hit the wheels of the main climber immediately behind, and not launch a pressure wave that increases on the way to the ground. If this lightly loaded mini-climber did as expected and stopped with 50 m/s^2 acceleration, it would put only a kilonewton of stress on the tether.

Mitigation 2: The ground node can have an agile acoustic transmitter and unreeling mechanism that releases tether to match the incoming shock wave.

Mitigation 3: Acoustic climbers with magnetread "wheels" (sec. 6) will be agile, and can jump over protruding defects in the tether. The magnetread will be stronger and have much less inertia, so it is unlikely to fail as easily as a pinch wheel.

Other mitigations may be possible. Approaching GEO, a magnetread climber may be able to climb at a much higher speed, but until we are sure it will always fail safely, that creates too much risk for the tether.

6 Magnet Tread Wheel

Sometimes, reinventing the wheel is unavoidable.

The tether, from ground to GEO, is almost 36,000 kilometers high, nearly the diameter of the Earth. That is a long way to travel on wheels; a terrestrial road vehicle travelling at 100 kilometers per hour would need 15 days to travel that distance. Except that this "road" starts straight up the side of the gravitational hill; the "slope" of the hill reduces to the equivalent of 45deg at 1200 kilometers altitude. Halfway to GEO, the equivalent slope is 3.2deg. Obviously, a land vehicle could travel a lot faster on the gentle slope than the steep one - with the right wheels!

A Concorde at 18 km altitude, flying at Mach 2 (2140 km/h or 600 m/s), could (with enough fuel) travel 36,000 km in 17 hours. The Concorde's wheels couldn't turn nearly that fast; it took off at 100 m/s and landed at 75 m/s. Once it did not ... Air France 4590 crashed after a wheel-related failure during (horizontal) takeoff. The Concorde suffered from 30 times more wheel damage than subsonic airliners. Compliant tires rotating at 100 m/s on a horizontal surface are at the edge of practical limits.

Maglev trains can go faster than 600 km/hour (nearly 170 m/s) because they travel on magnetic fields over conductive tracks. They don't go up steep slopes, but they do encounter high air friction at these speeds, through air ten times denser than the Concorde flew through.

Space elevator vehicles should climb as fast as possible, minimizing their added load as quickly as possible. The optimum climber "wheel" would turn very fast under considerable loading. Could we replace wheels with magnetic levitation?

Not directly; if the thin tether is conductive, it will not be much more conductive than a thin sheet of aluminum foil, so it will not emulate the thick high-traction steel roadbeds used

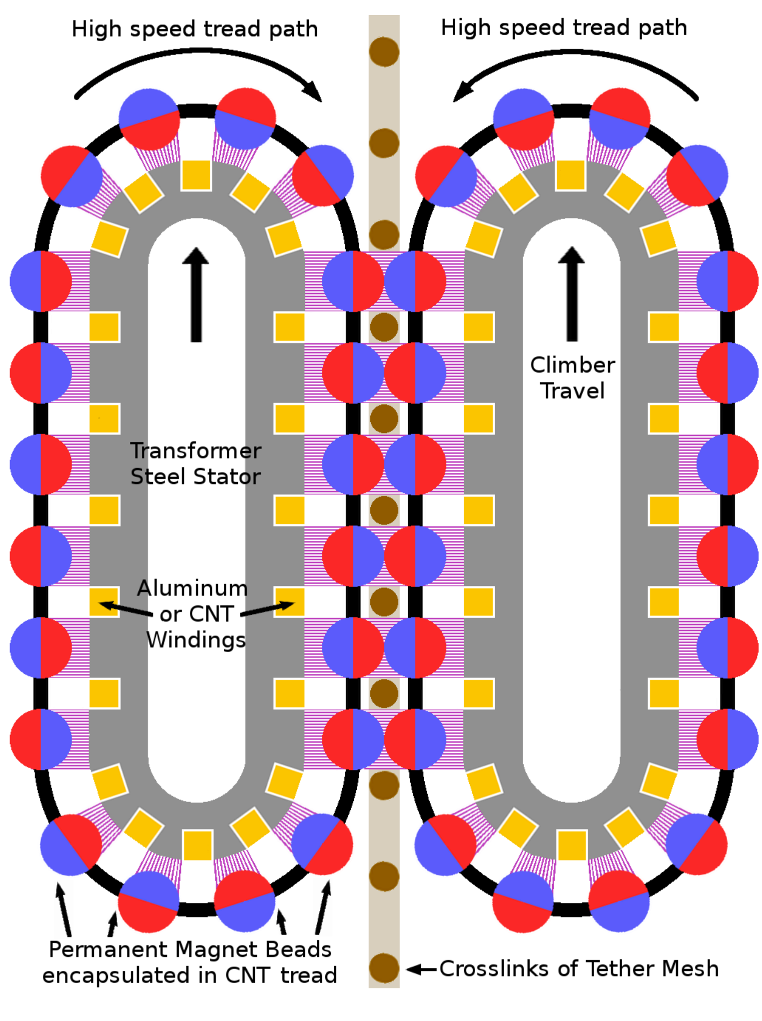

Figure 4: **Magnetread climber "wheels"**

by maglev trains. However, slippery roadways are nothing new; the solution, invented over a century ago, resembles the continuous or "caterpillar" track, also known as a tank tread.

We don't have deep compliant earth to dig into, but something better; access to both sides of a meshlike tether, a grid of holes our "tread" can grapple through. We don't want to damage the mesh with metal pins. Instead, we can build our opposing treads with shaped magnets that attract each other through the holes.

Figure 4 is a schematic illustration of two opposing treads with 20 round magnetic beads. These might be like beads of a necklace, with holes through the magnets that CNT threads pass through. The blue and red halves represent north and south magnetic poles, which attract each other and "pinch" through the mesh of the tether. The beads will also be in CNT fabric pouches, to contain the fragments if the beads shatter. The bead shape will be more complex than shown.

The magnets will be suspended in a magnetic field from a steel stator with aluminum (or perhaps high conductivity forms of CNT, when those materials become available). The stator can maintain perhaps a five millimeter spacing to the magnets with electronic control tuned by precision measurement. Similar controls maintain head height to nanometers in fast rotating hard drives; there are well understood solutions.

Actual treads will be much longer and have many more small magnets than shown. If the area density of the magnet tread is 45 kg/m^2, then a 1.5 Tesla magnetic field can deflect the tread with 20,000 m/s^{2}acceleration. If the tread travels at 200 m/s, the "D magnets" at the ends can deflect the magnetic tread around a 2 meter radius.

The fixed magnets do not spin, only the magnetic field does. if the magnets are spaced 2 centimeters apart (4 centimeters between "north and north") then 200 m/s is a 5 kHz field rate, a lower frequency than most electronic switching supplies (50 to 1000 kHz). Switching speed is not a problem, though the laminations for the stator must be thin to reduce eddy current losses and heating. Given the open geometry, this will be easier than a circular motor, and no electrical or mechanical contact is needed to the rotor or "Magnetread" when it comes up to speed.

The space elevator tether will have holes and broken vertical threads; a likely response will be necking at the sides of the holes, which will displace the bands of vertical holes over kilometer scale distances. So, the tread shown will actually be multiple parallel treads, thin bands treading down every other row of mesh holes on the tether. That allows a magnetread band to displace sideways on the tether.

The upbound wheel lock problem is easy to avoid with actively controlled magnets paired in bands. Referring to fig. 4, the bands of magnets are shown with opposite poles attracting. With the individual magnets flying past the transformer stators at with a 100μs spacing in time, delaying the magnets on one side by 50μs and advancing the magnets on the other side by 50μs would place them "like to like", they will repel each other, drawing closer to the stator poles, which will also be pulled away with strong springs. Like the SawStop table saw, it should be possible to deflect and withdraw a pair of magnet bands from the danger zone in milliseconds.

Worst case, one of the bands breaks. Above and below around the ends of the wheel will be CNT catcher buckets that can capture the thrown tread. There will be perhaps two dozen magnetread bands per "tread", and multiple treads per climber; with agile electronic control of the position of each magnet bead, and real time computation and mechanical modelling of the beads, bands, and tether ahead, the magnetread can optimize the position of every magnet bead.

We live in a precision age. We can position masking systems in semiconductor imaging systems to nanometers. The Large Hadron Collider beamline positions of 5,600 bunches of protons to 1 micrometer spacing around a 4 kilometer diameter tunnel. The proton bunches travel at 0.999999991 times the speed of light, and contain as much total kinetic energy as a 400 tonne TGV bullet train travelling at 200 km/h. Compared to the LHC, a 1000 m/s magnetread is standing still.

Why the high speeds? There are three critical zones for the space elevator tether.

The initial climb zone will be slow. The amount of tension the lowest vehicle can put on the tether (combined weight and acoustic power tension) is limited by the tension of the next vehicle up. The faster this previous vehicle can climb, the sooner it is "out of the way" and the lowest vehicle can accelerate to its top speed. Smaller, faster vehicles can greatly increase throughput.

Above that is the passage through the van Allen radiation belts, the province of the previous and "previous previous" climbers. The faster the vehicles go through these belts, the lower the accumulated radiation damage. Apollo travelled to the Moon at much higher speed, in an inclined orbit, so moon-bound astronauts spent only a few minutes travelling through the fringes of the belt. Space elevator climbers will spend many hours in the thickest regions of the belt; fast wheels and high power will be essential for survival.

Above that is the long journey to GEO. The "slope" is gentle, the mass penalty is small, so the staged climbers can be relatively heavy, with very large (though gossamer) magnet treads. With thinner magnet treads, and much larger radii, 500 m/s climb may be possible.

To an electronics engineer, 500 m/s is 0.5 micrometers per nanosecond. A nanosecond is enough time for a centimeter scale distance measurement, or 11,000 floating point multiply-adds by an nVidia Pascal GP102 video card. We can do an insane amount of measurement, computing, simulation, optimization, and control at the "high" speeds that a magnetread can turn. Climbers may lift payload to GEO, but they will also generate terabytes of data on every trip, helping engineers optimize the trips that follow, detect imminent failures before they happen, and correlate those potential failures all the way back to the raw materials individual magnetread beads were made from. Six sigma design will be essential to the long term survival of the space elevator tether, and its profitable operation.

Existing permanent materials like Neodymium-Iron-Boron are strong enough for a magnetread bead, but fragile and rather expensive. Alternate "earth abundant materials" are now being developed, first at microcircuit scale and later for electric car motors. Nano-structured iron nitride magnets are theoretically twice as strong as NIB magnets, and the source materials are pennies per kilogram. When we learn how to cheaply make kilotonnes of nano-structured carbon nanotube (or diamond!) tethers, iron nitride (or other promising earth-abundant materials) will be cheap, too.

7 Reflections, Receivers, and Transmitters

The receiver converts acoustic power - stress and velocity waves - into vertical thrust and velocity. The process resembles the capture of electrical energy with an antenna, resonant circuit, and rectifier.

Fig. 5 shows elements of an acoustic power receiver. For 100% power extraction, the reflection motor absorbs force and reflects all cable velocity back to the power extraction motor, creating a half-wavelength-delayed reflection back to the extraction motor. The sinusoidal force absorbed in the extraction motor will be in phase with the velocity, and can extract all the velocity in the wave. With 100% absorption, the reflection motor absorbs no acoustic power, though it will create considerable force, consuming power in the drive windings.

A climber can extract climb power transmitted from above, below, or both (if the wavelengths are compatible). Eventually, upper stage climbers will be scaled up to large, gossamer, and fast, with power from above like figure 5A.

Initially, the space elevator will generate acoustic power only from a terrestrial power sources (figure 5B) and used mostly by the power hungry climbers near the ground.

A simple acoustic climber is built around a power extraction motor facing the acoustic power source and the incoming acoustic power waves, and a reflection motor creating a resonant region between (note: the "motors" will probably be magnetic treads, described in section 6). In the simplest case, 100% of the tether acoustic power is removed by the power

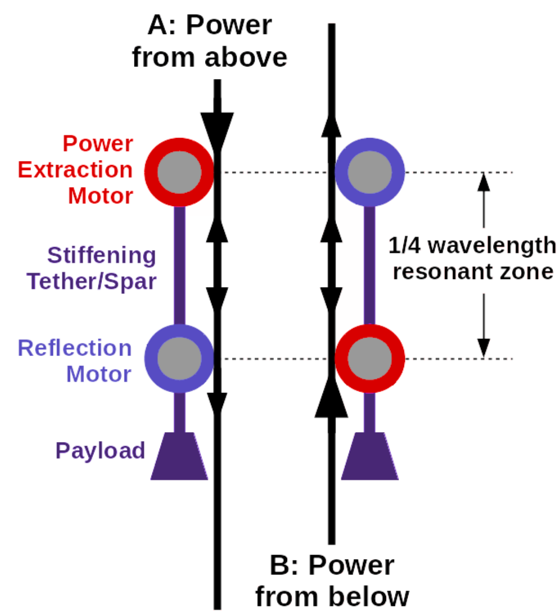

Figure 5: **Acoustic receivers**
A: Power captured from above, driving the power extraction motor and pulling up on the stiffening tether, is more effective and lightweight than ... **B:** power captured from below, which must be transmitted up the tension spar to the reflection motor above, which does the lifting in order to keep the spar under tension.

extraction motor. The reflection motor reflects a
standing wave resonance back to the power extraction motor, and stops acoustic energy (velocity and force) from propagating further along the cable.

The tether is continuous and stiff, so the velocity must the same on both sides of a motor. Only the force can change, with the motor generating (or absorbing) the force. Over quarter-wavelength distances, tether velocity and tension can change considerably, and we will exploit that to produce a resonance.

The reflection motor reflects the power by creating an equal and opposite force to counteract the partial wave passed by the power extraction motor, 1/4 of a cycle ago. If the climber was stationary, the reflection could be provided by a fixed attachment.

The opposing force is a reflection that arrives at the power extraction 1/4 of a cycle later. The half-cycle delay means that the reflected wave arrives at the power extraction motor 180° out of phase with the wave arriving from the source. The reflection motor inverts the velocity and not the force, but the half cycle delay inverts both. The net effect is that the displacement velocity on the inside of the power extraction motor matches the arriving acoustic wave velocity, while the acoustic displacement force just behind the extraction motor sums to zero, for an acoustic climber absorbing 100% of the incoming acoustic power.

The velocity times the force difference is power that is extracted by the power extraction motor.

Figure 6 illustrates the process. The column down the right shows the displacement force and displacement velocity along the tether between the power motor and the reflection motor, 200 milliseconds or 72° apart for this 6 frame "movie", with the incoming power (displacement velocity and displacement force) moving towards the right.

This illustration shows a simplified situation; the tether acoustic conductivity G is 400 kg/s, incoming power (coming from the left) averages 2 MW, the maximum displacement force is ± 40 kN, and the maximum displacement velocity is ± 100 m/s. At 28 km/s and one hertz frequency, the wavelength is 28 kilometers, and the motors are 7 (!) kilometers apart, connected by a "stiff" (relative to the tether) spar. The tether linear mass density $\bar{m}$ is 400 kg/s divided by the acoustic velocity of 28 km/s, or 14.3 kg/km; with a density of 1300 kg/m^3, the cross section of the tether is 11 mm^2. That would be 185 kilometers altitude on the assessment tether; not a significant altitude, besides making the numbers easy to compute ...

The graph on the lower left of figure 6 shows the motor velocities versus time. For this simplified situation, the climber is in a fixed position, with the 2 MW of absorbed acoustic power not being used for climb.

The force on each motor, which is the *difference* between the incoming and outgoing acoustic wave forces, is shown in the graph above the lower left. Again, normally the motors will be dividing the burden of supporting and lifting the climber, and use power to do so.

The acoustic power extracted by (or generated by) a motor is the velocity multiplied by the force: $P = FV$, shown in the third graph up from the bottom left of fig. 6. The reflection motor has zero acoustic velocity, so the it extracts no power. The power extraction motor extracts power varying sinusoidally at twice the wave frequency: ($\sin(f) \times \sin(f) = \frac{1}{2}(1 - \cos(2f))$).

The power from the power extraction motor is sinusoidal, averaging 2 MW. For a smooth ride, the peaks of the extracted power will be stored in capacitors, or perhaps in a mechanical resonance in the climber, and fill in the valleys, so a steady 2 MW of climb power is available. For a 16 tonne vehicle in the 9.2 m/s^{2}gravity field at 185 km, that is enough power to climb at 13.6 m/s, or 49 km/h.

Acoustic receiver motors will have significant mass and inertia. The inertia of the motor (which is "capacitance" in our electrical/mechanical analogy) plus the compliance of a shorter-than-quarter-wave length of tether can be tuned as a resonator, "tuning out" the motor inertia . This resembles a capacitor-stubbed antenna match network for a long wavelength radio antenna. Short radio antennas transmit and receive radio energy less effectively, but the tether confines the acoustic energy, so a "stubbed" acoustic receiver won't be handicapped like the radio antenna.

However, for this discussion, impedance matching and the complex-plane phase diagrams are an unnecessary complication. Let's assume massless motors and analytically simpler quarter-wavelength acoustic receivers, leaving the complexities to the clever engineers paid to solve them.

The acoustic receiver just described has a reflection coefficient r of 1.0, indicated with the arrow on the top left graph of fig. 6. The rest of the graph is fascinating, and we will use fig. 7 to understand it.

Figure 7 is complicated, but helps explain how quarter-wave acoustic climbers operate. Acoustic waves can propagate to the left or to the right, or both. A wave propagating purely to the right has a displacement force equal to the acoustic conductance times the displacement velocity, $f = Gv$. The displacement velocity

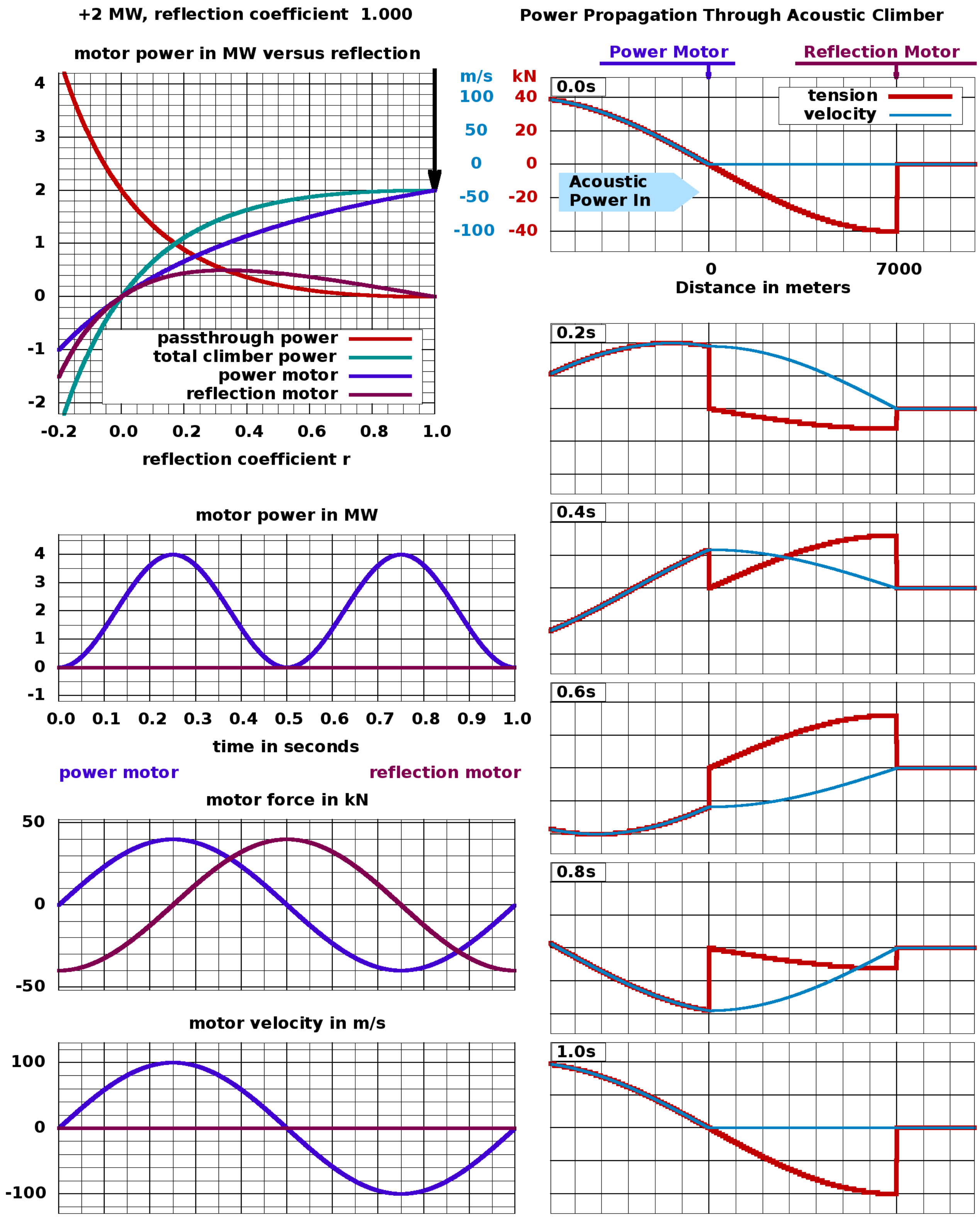

Figure 6: 2 MW power absorbed; using all the power arriving at the climber.

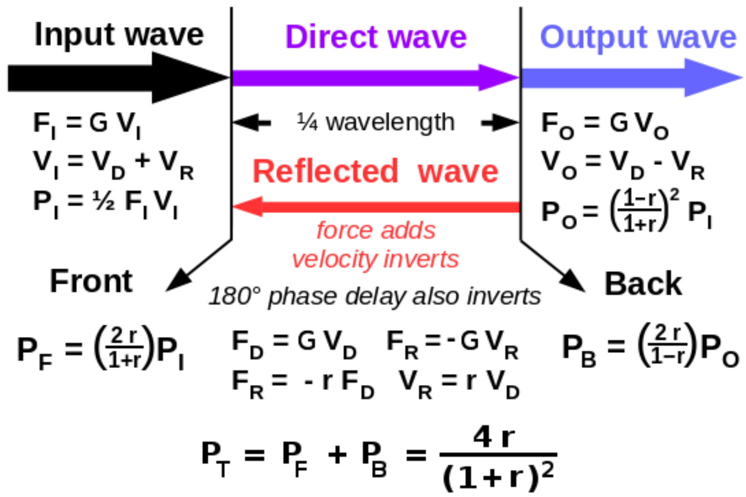

Figure 7: **Acoustic receiver velocity, force and power**

v, the "wiggle" from the static middle, is much smaller than the propagation velocity c, the rate at which the displacement force and velocity force propagate left or right on the diagram.

If the wave is propagating in the opposite direction, purely to the left, the displacement force is negative; $f = -Gv$. The acoustic conductance G is the mass per length $\bar{m}$ times the acoustic propagation velocity c, $G = \bar{m}c$.

Generally, there can be acoustic waves travelling in **both** directions. Because the acoustic receiver will be imperfect, it may reflect some power back towards the source. If the reverse wave is the same amplitude as the forward wave, this is called a standing wave, and the tether will oscillate in place, packets of velocity and tension appearing to bounce back and forth against each other, zero power transmitted. More often, one wave dominates, and the amplitude versus distance will have maximums and minimums. The maximums add unnecessary tether strain without moving useful power.

The ratio of maximum to minimum is the **standing wave ratio** or **SWR**, which should be monitored continuously. Acoustic receiver forces should be continuously tuned to make the SWR unity.

The more general case that both motors can absorb or (generate!) power, depending on the value of the reflection coefficient r. A generator is a more versatile motor. So we will generalize our descriptions as well; the power motor becomes the "front" motor, and the reflection motor becomes the "back" motor. We will consider four waves: the input wave from the source upstream, the output wave downstream from the climber, and in between a direct wave and a reflected wave in the 1/4 wavelength resonator between. The resonator is intentionally designed to have a standing wave. The amplitude of the wave may be zero at the back motor, so the standing wave ratio would be infinite; not a useful concept for the discussion that follows.

The displacement velocity must be continous at front and back motors, as mentioned before. That means that the peak displacement velocity of the input wave, v_I, must equal the sum of the direct wave v_D and the reflected wave v_R. The back motor is designated the reflector because we will characterize it with a reflection coefficient r. Choosing the back motor as the reflection point is is an arbitrary choice; we can also analyze the system going the other way. Our pair of motor/generators will be used both ways. At the back motor, the the reflection subtracts from the output wave, and is a fraction r of the direct wave:

$$v_I = v_D + v_R \quad (8.1) \qquad v_O = v_D - v_R \quad (8.2) \qquad v_R = r \times v_D \quad (8.3) \tag{8}$$

From those three equations, we can compute the values of v_D, v_R, and v_O as a function of r:

$$v_D = \frac{1}{1+r} v_I \quad (9.1) \qquad v_R = \frac{r}{1+r} v_I \quad (9.2) \qquad v_O = \frac{1-r}{1+r} v_I \quad (9.3) \tag{9}$$

All the waves are associated with displacement forces, $F = \pm GV$, with the reflected wave going backwards and the peak force "negative". The round trip over half a wavelength inverts both displacement velocity and force.

$$f_I = G v_I \tag{10}$$

$$f_D = \frac{1}{1+r} f_I \quad (11.1) \qquad f_R = \frac{-r}{1+r} f_I \quad (11.2) \qquad f_O = \frac{1-r}{1+r} f_I \quad (11.3) \tag{11}$$

The displacement forces at the front and back motors, f_F and f_B, are the difference of the sums of the left and right forces:

$$f_F = f_I - (f_D + f_R) = \frac{2r}{1+r} f_I \quad (12.1) \qquad f_B = f_D - (f_R + f_O) = \frac{2r}{1+r} f_I \quad (12.2) \tag{12}$$

The peak power at the input, output, and extracted from the front and back motors, is the displacement force times the displacement velocity. The average power P (in this stationary example) is half of that:

$$P_I = \frac{1}{2} f_I v_I \quad (13.1) \qquad P_O = \frac{1}{2} f_0 v_O = \left(\frac{1-r}{1+r}\right)^2 P_I \quad (13.2) \tag{13}$$

$$P_F = \left(\frac{2r}{1+r}\right) P_I \quad (14.1) \qquad P_B = \left(\frac{2r(1-r)}{(1+r)^2}\right) P_I \quad (14.2) \qquad P_B = \left(\frac{2r}{1-r}\right) P_O \quad (14.3) \tag{14}$$

The total power extracted from both motors is:

$$P_T = \left(\frac{4r}{(1+r)^2}\right) P_I \tag{15}$$

If the reflection coefficent is set to one, then $P_T = P_I$ and $P_O = 0$, which is our original, simple case. In the general case, a climber may extract a fraction $T = P_T/P_I$ of the power with a reflection coefficient of:

$$r = \frac{1 - \sqrt{1-T}}{1 + \sqrt{1+T}} \tag{16}$$

The tap can extract half the power ($T = 0.5$) with $r = 0.24847$, which is illustrated in fig. 8.

If T is **negative** 0.5 , then $r =$ -0.13165, which is illustrated in fig. 9. The acoustic motor pair actually **adds** 1 MW to the 2 MW input, to output **3** MW.

Descending climbers must somehow dissipate gravitational descent energy. By depositing energy through the same mechanism, on the "downbeat" of the wave, a descending climber can transmit power to an ascending climber with a payload, requiring less power from distant ground or GEO transmitters.

The first job of the space elevator is lifting revenue-generating payload, but there will be gaps in the manifest, so there will be capacity to lift acoustic transmitters and power sources to GEO, eventually to power mid and high stage climbers. A 1 Hz transmitter could be mostly mechanical, perhaps a solar-powered rotary heat engine providing the bulk of the power through an acoustic transmitter "tuner".

The acoustic propagation delay from GEO to High Stage One is more than 20 minutes, resulting in a long delay and potentially high attenuation between a GEO acoustic power transmitter and power pick-off by lower climbers.

Our attention (and the bulk of the power) should be focused on the power-hungry near-ground climbers, mostly powered from the ground. Lifting a kilogram to GEO with a space elevator requires 52 MJ. If we

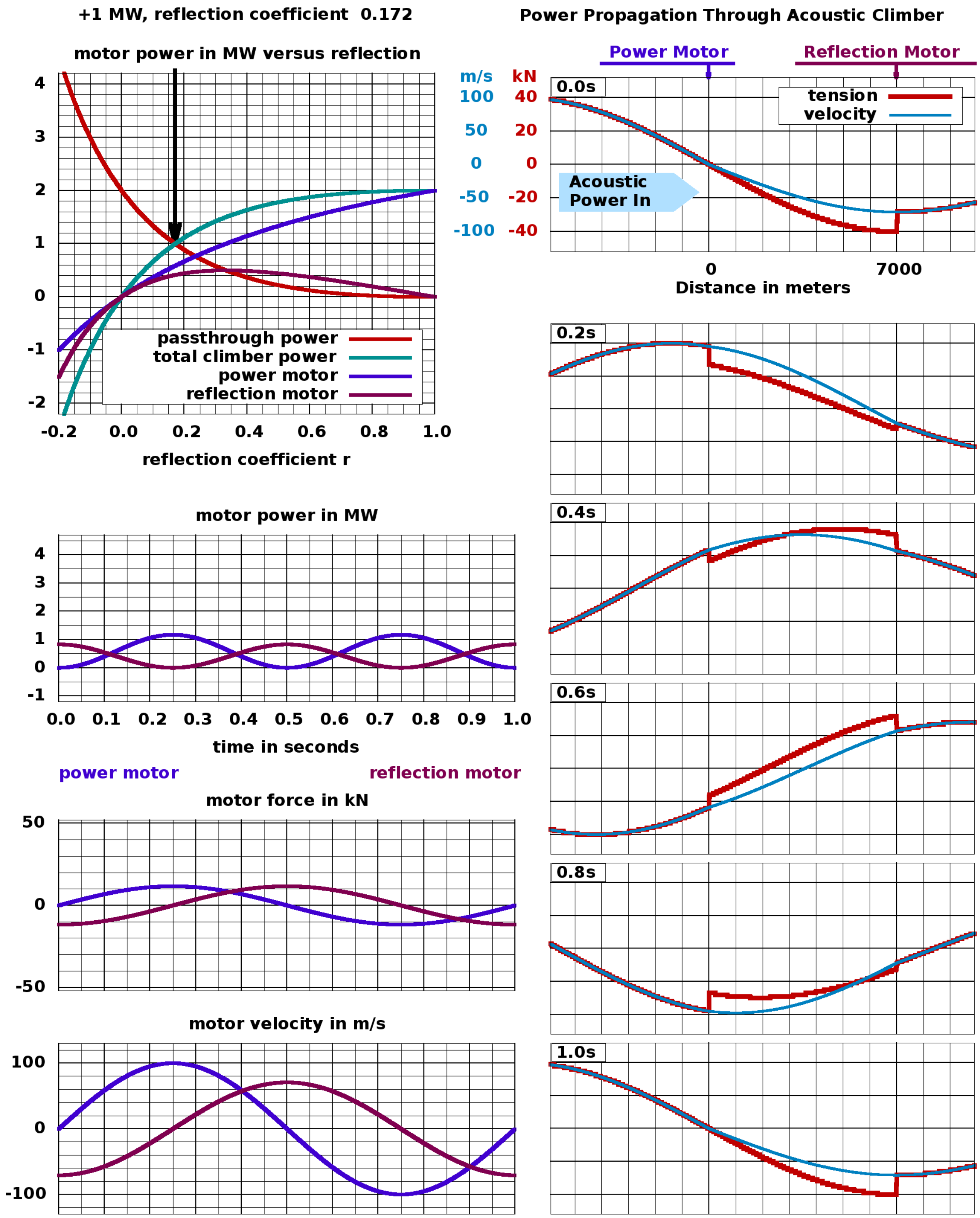

Figure 8: 1 MW power absorbed; using half of the power arriving at the climber.

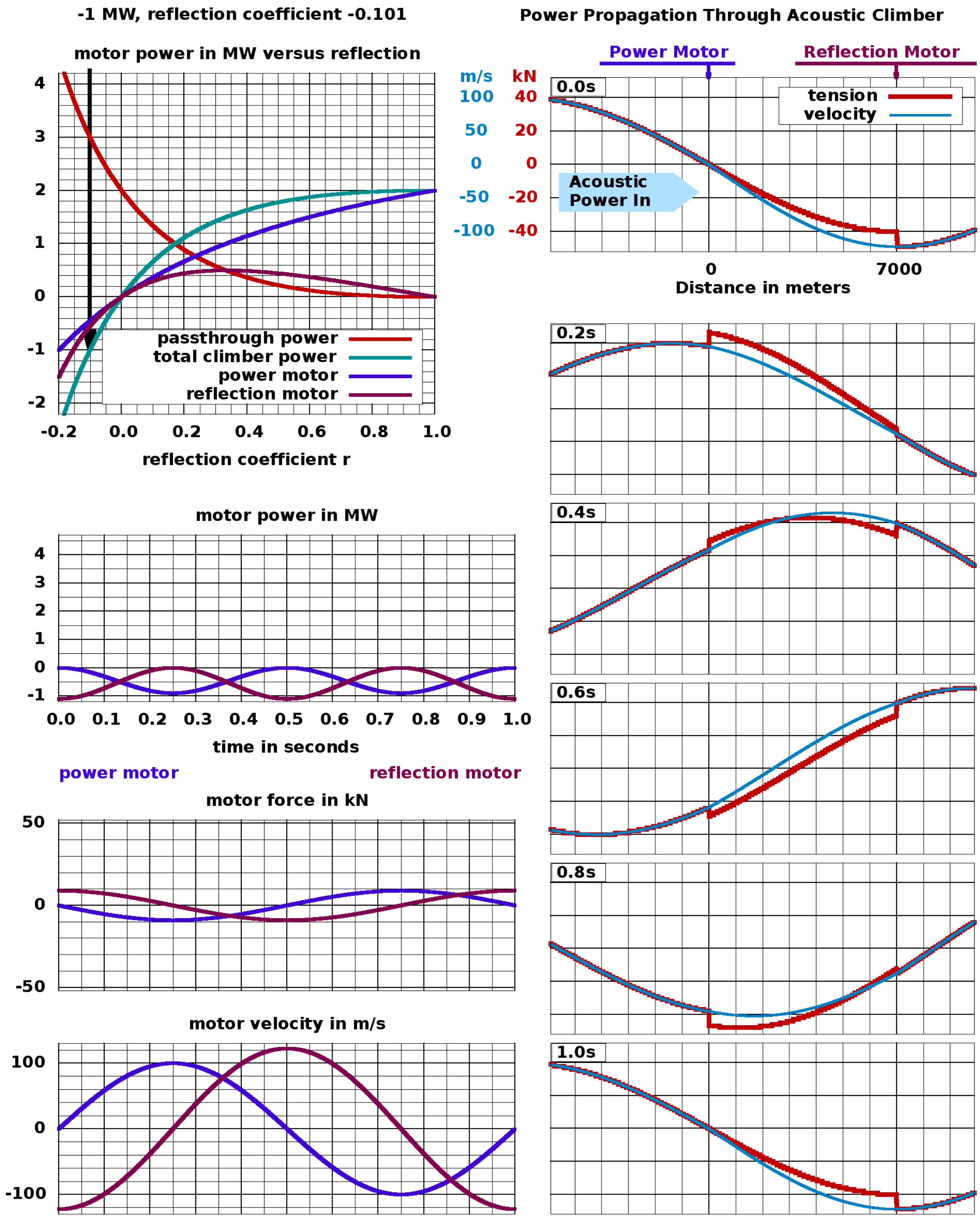

Figure 9: 1 MW power **generated**. Adding 1MW of power to the tether, shedding climber descent energy.

lift 14 tonnes per 24 hour day, and recycle half the descent energy of returning 2 tonne climbers, then (for calculating power) the effective mass flow is 15 tonnes upwards per day, 625 kilograms per hour, or 0.174 kilograms per second, an average power level of 9 MW.

Assuming inefficiencies, a 20 MW electric generator is adequate. At 20 cents per kilowatt hour, that is $35 M/year, small compared to other expenses, so efficiency is not the main consideration.

Acoustic power frequencies should be chosen for optimal climber performance and reliability.

It would be convenient if the frequencies were high and wavelengths short. That would make the receiver spar short, and reduce the amount of climb energy that must be stored over a cycle.

However, high frequencies mean more hysteresis, and motor accelerations will be rapid, requiring more power to overcome their inertia.

The operating frequency will be chosen to optimize tether hysteresis response and maximize power transmission, which depends on material properties we cannot predict at this time.

If the frequency is 1 Hz, and the acoustic velocity is 28 km/s, then a wavelength is 28 km and the 35768 kilometer long tether between the ground and GEO nodes is 1278 wavelengths long. 0.1% hysteresis loss per cycle would dissipate 72% of the power transmitted from GEO before it reaches climbers near the ground. OTOH, if hysteresis losses are less than 1 ppm (as per-climber-cycle creep must be to prevent long term stretch), then the power transmission approaches 99% for a 100 Hz vibration frequency and 128,000 cycles on the tether.

Table 2 assumes a 16 tonne climber climbing at 100 m/s and close to the geostationary station at 35786 kilometers altitude. 5 to 15 climbers between 5000 km altitude and GEO will tap off only a small fraction of the acoustic energy sent from GEO, passing most of it to climbers below.

6 MW tether, 80 g/m near GEO			
conductance G	2240 kg/s		
vibration energy	214 J/m		
displacement velocity	73 m/s		
displacement force	164 kN		
static force	2160 kN		
local gravity	near zero		
vehicle mass	16000 kg		
payload mass	14000 kg		
climber mass	2000 kg		
vehicle speed	100 m/s		
climber support force	near zero		
climber climb power	near zero		
propagation speed (c)	28 km/s		
frequency (Hz)	1	10	100
period (ms)	1000	100	10
1/4 wavelength (m)	7000	700	70
energy per cycle (kJ)	6000	600	60
displacement (cm)	1160	116	11.6
acceleration (m/s^2)	460	4600	46000

Table 2: **Acoustic powered climber near GEO** Higher frequencies allows shorter climbers, but motor acceleration (and related inertial effects) are higher, and tether hysteresis losses will be higher as well.

The acoustic transmitter at the ground will be much like a climber, operating between the ground node bottom of the tether and a storage spool or a splicing station.

The GEO and anchor nodes will also be climbers, slow ones. Soon after the beginning of cargo operations, powered only from the ground, some of the first cargos delivered to GEO will be a multimegawatt PV array. This will be standard comsat power technology, photovoltaic panels designed for microgravity and not for the high gravity futher down the well, unfurled and spread around slowly rotating pivots to track the Sun. These nodes will be eclipsed once a day in the weeks around the equinoxes. They might have large, heavy storage batteries to supply climb power during these 70 minute (maximum) interruptions. Otherwise, this is a good time to schedule transfers between staged climbers.

Transmitter motors provide another advantage with an inclined tether; they can rapidly change the tether tension, and hence its north-south profile versus altitude. This allows the tether to dodge accurately tracked space debris, given some warning. A debris object at 1000 kilometers altitude can be avoided with a large vertical displacement wave launched from the lower transmitter 40 seconds before.

The general flexibility of acoustic motor climbers will greatly enhance the productivity and safety of the space elevator tether.

8 Acoustic Climb to GEO

The space elevator tether is a challenging logistics environment.

Vehicles on the lower tether require huge amounts of externally-supplied power to climb against full gravitational forces (9.76 m/s^{2}at the surface), while vehicles halfway up to GEO climb against 0.55 m/s^{2}acceleration, 18 times less. The upper half of the tether accounts for almost half the ground to GEO travel time, but climbers consume relatively little power.

Climber descent for re-use must dissipate the climb energy, either radiating it as heat from a filament, broadcasting it as UVLF power to scatter radiation belt particles, or transmitting it back down the tether to help power ascending vehicles.

At the surface, a 16 tonne acoustic elevator vehicle (14 tonnes payload, 2 tonnes acoustic climber) adds 156 kN of force to the tether above. An additional 4 tonnes of mass would increase the force to 196 kN, another 40 kN.

Starting with 2.5 MW of acoustic power P propagated from below, through the $G = 382$ kg/s conductance of the tether near the surface, there will be a peak acoustic force of $f_a = \sqrt{2PG} = \sqrt{2 \times 2.5e6 \times 382} = \pm 44$ kN on the tether.

The tether beneath the first vehicle must always have positive stress on it (you can't push a rope!), or it will not pass acoustic power, or it may vibrate transversely (side-to-side), destructively. So, the average stress beneath the vehicle must be more than 44 kN; as a wild guess, at least 60 kN. The initial vehicle weight added to the average initial stress on the tether below is 216 kN, the weight of 22 tonnes of mass.

Let $F_b = f_a/k$ be the additional stress beneath the vehicle. The total stress is $F_t = f_a/k + F_v$. F_t is a function of the tether, and changes with altitude, complicated by taper, stretch, safety factors, and the mass of the tether below. For the following analysis, we will pessimistically assume that F_t is constant.

2.5 MW is enough power to lift the vehicle at 16 m/s; $v = P/(Mg) =$ 2.5e6 / (16000 × 9.76) = 16.0 m/s. At this climb rate, the gravitational force on the vehicle diminishes by 0.78 newtons every second, which allows the vehicle to accelerate. As the vehicle climbs at velocity v, gravity g (including centrifugal acceleration, small near the surface) changes over time.

$$\dot{g} = \frac{d}{dt}\frac{\mu}{r^2} = -2\frac{\mu}{r^3}\frac{dr}{dt} = \frac{-2gv}{r} \tag{17}$$

The reduced gravitational force F_v on the vehicle allows the stress f_a to increase; $\partial f_a = -k\partial F_v$. As a function of time, that is $\dot{f}_a = -k\dot{F}_v$.

The vehicle climb power is $P = Mvg$; rearranging, velocity $v = \dot{r} = P/Mg$. More acoustic power, reduced gravity with increasing altitude.

What is the acceleration $\ddot{r} = \dot{v}$? $\ddot{r}$ can be decomposed into changes due to increasing power $\dot{P}$ and decreasing gravity $\dot{g}$:

$$\ddot{r} = \frac{\dot{P}}{Mg} - \frac{P\dot{g}}{Mg^2} \tag{18}$$

The acoustic power $P = {f_a}^2/(2G)$, so

$$\dot{P} = \frac{2f_a\dot{f}_a}{2G} = \frac{f_a}{G}\dot{f}_a = v_a\dot{f}_a = -v_a k\dot{F}_v = -kv_a M\dot{g} \tag{19}$$

... because the peak acoustic velocity $v_a = f_a/G$.

We will ignore the effects of increasing G versus altitude on $\dot{P}$ for now; the math is very complicated and tether-design dependent, though we can roughly estimate the vertical variation of G as:

$$G = G_0\ exp\left(\frac{\mu_e}{Yu}\left(\frac{1}{r_e} - \frac{1}{r}\right)\right) \tag{20}$$

... not including centrifugal acceleration, design optimizations, or altitude-optimized tether materials and coatings (different for atmosphere, mesosphere, ionosphere, radiation belts, etc.), which will modify the specific strength Yu. Ambitious readers are welcome to extend the simplified analysis below to include these complexities.

time hours	accel. m/s^2	velocity m/s	altitude km	gee m/s^2	power MW	v_a m/s	f_a kN	G kg/s
0.0	5.0E-4	16.0	0.0	9.8	2.5	115	44	382
1.0	5.8E-4	17.8	61	9.6	2.7	119	46	389
2.0	6.7E-4	19.9	129	9.4	3.0	123	49	396
3.0	7.9E-4	22.3	205	9.2	3.3	127	52	404
4.0	9.2E-4	25.1	290	9.0	3.6	132	55	414
5.0	1.08E-3	28.5	387	8.7	4.0	137	58	424
6.0	1.28E-3	32.4	496	8.4	4.4	142	62	436
7.0	1.52E-3	37.0	621	8.1	4.8	146	66	450
8.0	1.82E-3	42.5	764	7.8	5.3	151	70	465
9.0	2.2E-3	49.0	928	7.5	5.9	156	75	483
10.0	2.7E-3	56.9	1119	7.1	6.5	160	81	503
11.0	3.2E-3	66.4	1341	6.7	7.1	165	86	526
12.0	3.9E-3	78.0	1601	6.3	7.8	168	93	552
13.0	4.9E-3	92.2	1908	5.8	8.6	171	100	583
14.0	6.0E-3	110	2271	5.3	9.3	174	108	619

Table 3: Acoustic vehicle acceleration; an oversimplified, illustrative example, inconsistent with the graphs below.

Substituting $\dot{P}$ and $\dot{g}$ into equation 18 we get find the vertical acceleration:

$$\ddot{r} = \frac{-kv_a M\dot{g}}{Mg} - \frac{Mvg\dot{g}}{Mg^2} = (kv_a + v)\frac{-\dot{g}}{g} = (kv_a + v)\frac{2v}{r} \tag{21}$$

The (very small) acceleration $\ddot{r}$ increases with vehicle velocity v and the larger acoustic velocity v_a. The increase is somewhat faster than a slow exponential.

At ground level, v_a = 44 kN / 382 kg/s = 114 m/s, kv_a = 84 m/s, v = 16 m/s (after a very gentle power ramp-up and acceleration), and r = 6.378e6 m, so $\ddot{r}$ starts out at 5e-4 m/s^2. The vehicle accelerates glacially slowly; after an hour, it is climbing at 17.8 m/s ... but the increased climb rate and acoustic velocity increase the $\ddot{r}$ acceleration to 5.8e-4 m/s^2.

The slow exponential increase in v_a, accompanied by an increase in power from the acoustic transmitter below, is approximated in table 3. The table is simplified, and uses a one hour time-step, which under-approximates acceleration, velocity, and altitude. This approximation also ignores limits on **maximum** acoustic tension, which may approach full tether capacity at high power. Vehicles above will also need power, though some may be supplied by an acoustic transmitter above at GEO, added months after initial operations begin.

With an acoustic velocity of 28 km/s, and nonzero hysteresis, the transmitted power may take as long as 20 minutes to reach a vehicle at the opposite end of the tether, and will be attenuated when it gets there. The acoustic transmitter must anticipate the power needed later, and climbers must adapt to unanticipated power surges caused by unpredictable failures and emergencies on climbers in between.

For the assessment tether, the optimal vehicle size is smaller than 16 tonnes, sent up the tether more frequently than once per day. The tether taper profile versus altitude may also be optimized for maximum throughput at lower altitudes. Acoustic climber technology, combined with vertical staging and climbers optimized for different velocity and gravity regimes, will evolve with experience.

Acoustic climber payload size and cadence can be irregular, adjusted for customer demand, and not tied to the 24 hour solar cycle. Acoustic climbers will have a long spar tether with a heavy high voltage power cable, but that can be coiled or folded, so climbers can carry other climbers, compacted.

Acoustic climbers intended for lower mass payloads lower on the cable can lift heavier payloads from higher altitudes. Larger payloads than 16 tons can be lifted to those higher altitudes with a heavier, slower bottom climber substituted for the normal-sized acoustic climber, though the operational cadence will be slower.

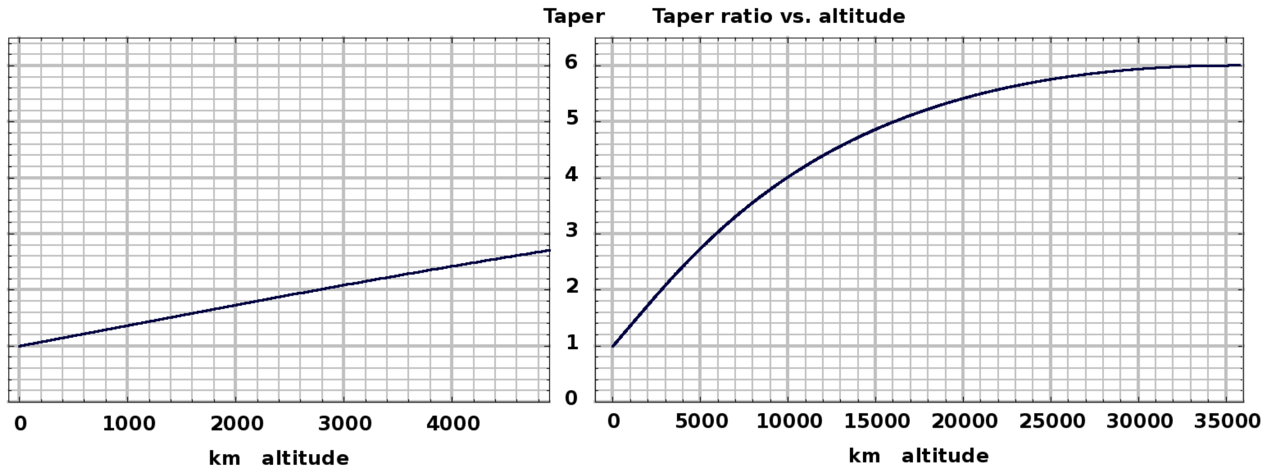

Figure 10: **Tether taper versus altitude.** Reduced gravity at higher altitudes reduces the rate of taper increase.

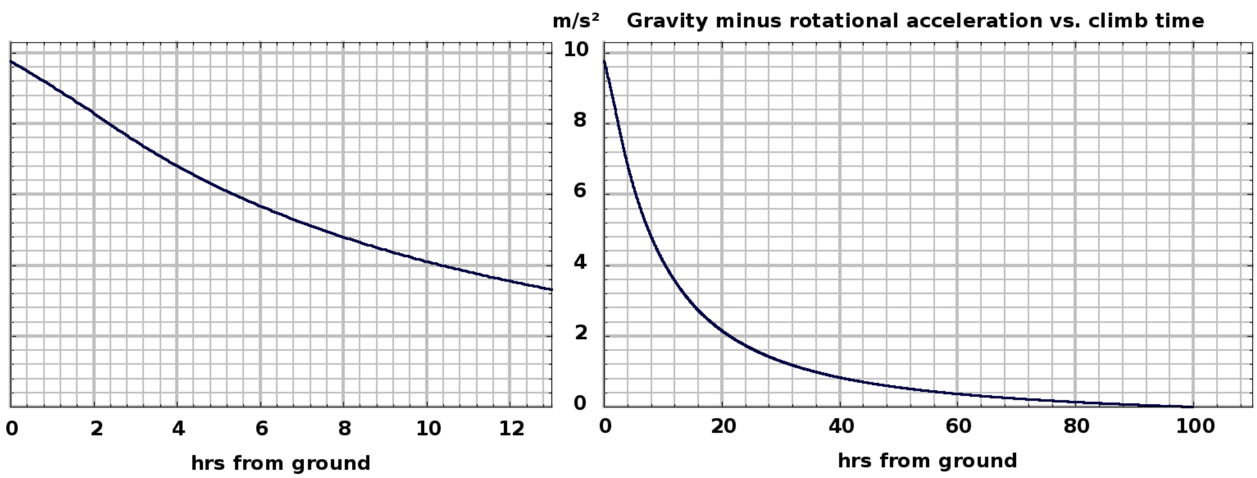

Figure 11: **Gravity (minus centripedal acceleration) versus altitude**

Figures 10 through 14 use the same assumptions as the table above, but were produced by a C language program [15] with more accurate numerical integration . The graphs show the climb of one individual climber, not many staged climbers, sharing power from a single acoustic transmitter at the ground node.

Figure 10 shows the taper versus altitude, and uses a simple model for taper cross section:

$$taper \;=\; taper_0 \;\exp\left(\frac{\Delta E(r)}{Y}\right)$$

$\Delta E(r)$ is the gravitational and rotational potential energy difference from the ground radius to the radius of the climber. Y is the specific strength of the tether in Yuris ($(\mathrm{m/s})^2$or $\mathrm{Pa}/(\mathrm{kg/m}^3)$), equal to the tensile strength divided by the mass density. A tether optimized for acoustic power and a particular vehicle mass and cadence may be less uniform than this relatively simple exponential function.

The energy difference between the halfway point at 17893 kilometers altitude and the ground node is 92% of the energy to GEO at 35786 km. The tether is 87% of the full taper at that altitude.

Gravitational minus centrifugal acceleration is shown in fig. 11. After 12 hours, the climb force drops and the acoustic power increases enough to permit 100 m/s climbs. If the vehicle mass was reduced by

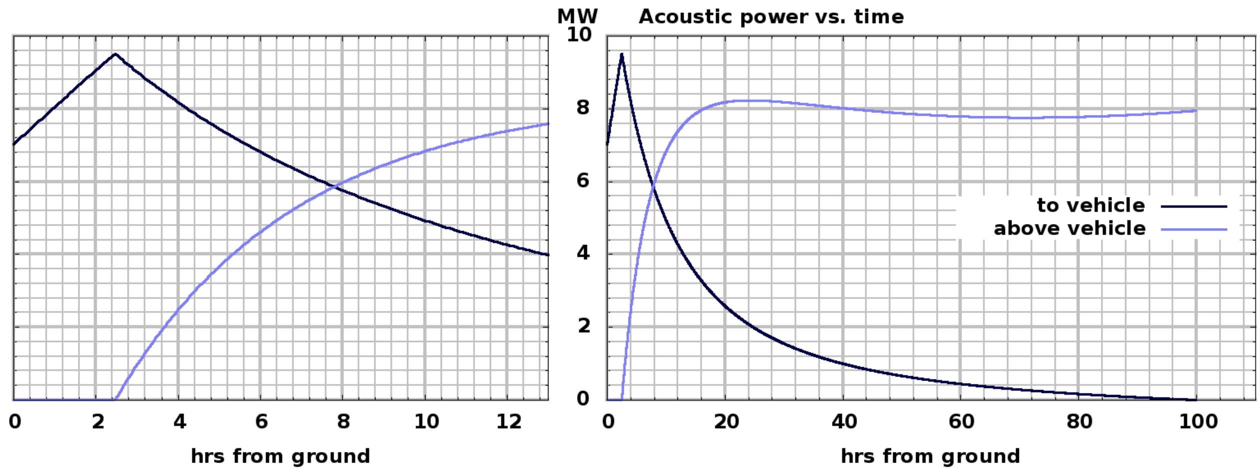

Figure 12: **Power vs. time, single vehicle on the tether** One vehicle on the tether. The assumed top speed is 100m/s for a 16 tonne vehicle. These numbers will increase greatly at middle altitudes when an acoustic transmitter is added at GEO.

perhaps 20%, there would be more remaining tension for acoustic power transmission, and the high speed climb could begin much sooner and closer to the ground. This, plus higher maximum speeds, could greatly increase tether throughput.

Vehicle power levels are shown in fig. 13. At the lowest altitudes, the gravitational weight of the vehicle limits the remaining tension available to transmit power. As the bottom vehicle moves higher, more of the tether's tension may be allocated for acoustic power, which is proportional to tension squared. For this paper, I assume vehicle speed is mechanically limited to 100 m/s (360 km/hr or 220 m.p.h.), as shown in fig. 13.

However, staged climbers, with payload hand-off from high-gravity low-speed climber designs below to low-gravity high-speed climber above, may improve top speed in the upper tether, and total throughput.

Section 6 suggests a "magnetread" climber, with embedded permanent magnet beads that grip tightly to each other through the weave of tether, and magnetically levitate against a long magnetic stator coil system. This should permit higher speeds. During a one hour handoff and payload exchange between climbers above and below, individual magnetic tracks will be peeled off and drawn back from the tether, inspected for wear, and replaced from spares aboard the climber.

Above 500 km (for this design), the acoustic power is more than ample to keep a vehicle climbing at 100 m/s, the assumed top speed. That suggests that another vehicle can start up the tether when this point is reached, passing some power upwards. However, the tether cannot support the weight of two vehicle this low in the gravity well. An optimized system may reduce payloads for most missions, maximizing total throughput.

With a lower transmitter only, the climb time for a single climber is shown in fig. 14.

Above 2000 kilometers, the tether is wider and gravity is lower. The tether can support many climbers, and the power needed for each climber to achieve 100 meters per second against the remaining gravity is lower, too, as shown in fig. Acoustic climber reflection magnetrack torque can be adjusted to pass most of the excess power to climbers above, and many climbers can use that power to climb at full speed.

9 Transfers between climbers

Different climbers will be optimized for different altitudes. If power is transmitted downwards, then upper climbers should efficiently pass most of the downwards power, while their gravitational weight is unimportant. They may be much heavier than lower climbers, to increase bypass power efficiency, power moved from the top motor to the bottom motor.

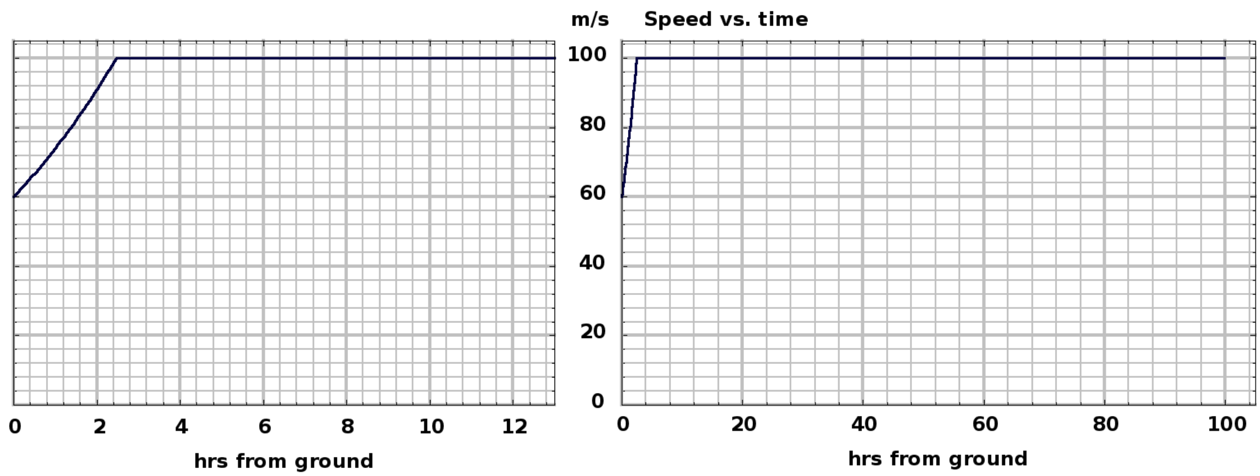

Figure 13: **Speed versus time, single vehicle on tether.** Speed is proportional to power divided by gravity and the 16 tonne vehicle mass. The maximum climb speed is presumed to be limited to 100 m/s. Below 400 km altitude, climb speed is power limited. Multiple vehicles on the tether reduces the surplus tension available beneath the vehicle to accomodate acoustic power.

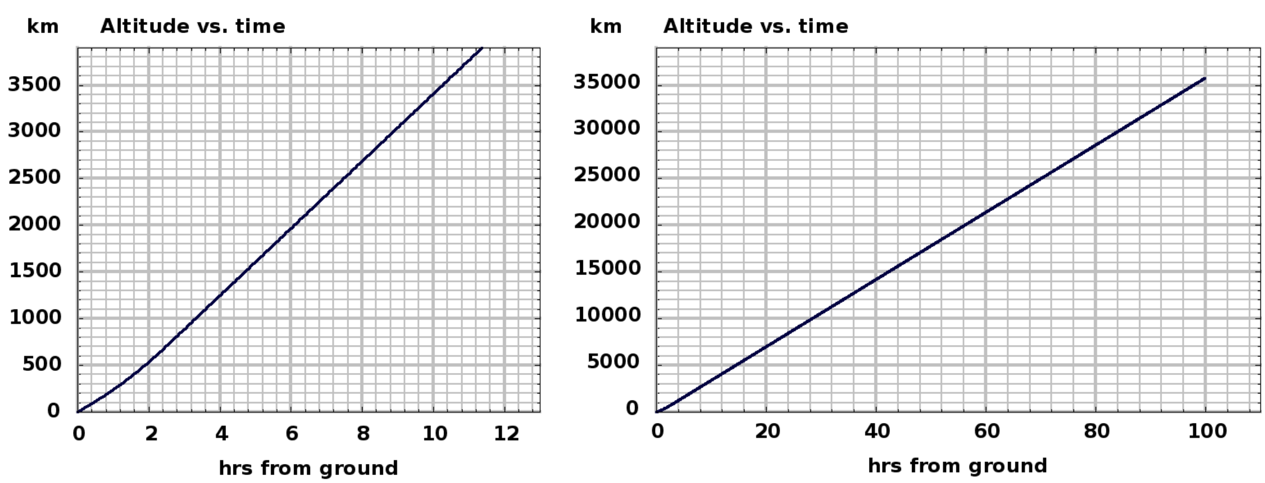

Figure 14: **Altitude versus climb time**. After 12 hours, the vehicle climbs at full speed.

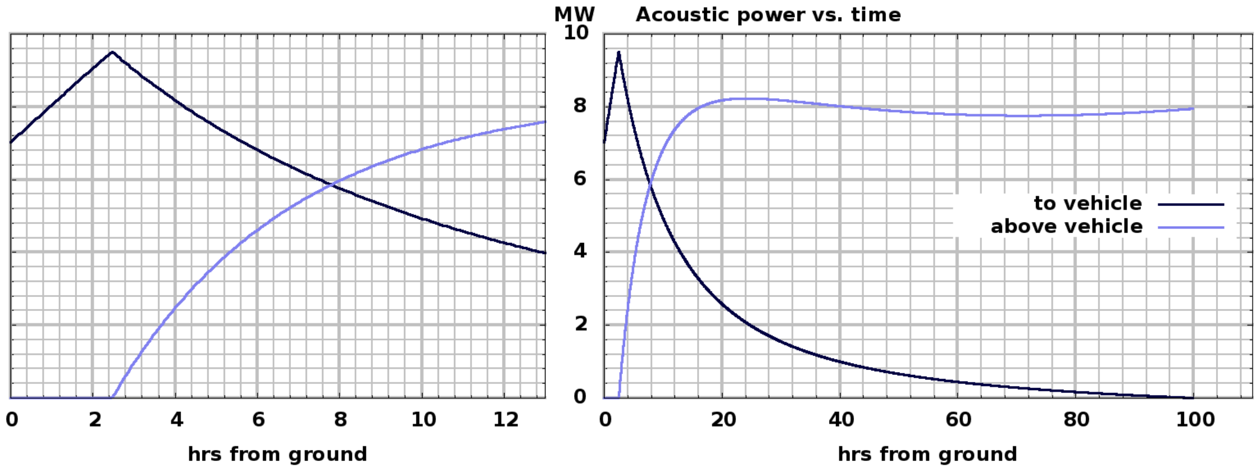

Figure 15: **Power used by a 16 tonne vehicle vs. time** Ascent speed limited to 100 m/s, or by available power.

Whatever their design, one climber will not travel all the way to the top, then all the way back down to the ground, nor is it efficient to stop operations for days while a convoy of returning climbers descends.

Instead, climbers will cycle up and down over relatively short distances [18] for 5 hours up and 5 hours down per 12 hour payload cadence period, at 100 meters per second speeds, 1800 vertical kilometers per cycle.

For tall solar-powered climbers, a handoff down past a stack of folded solar cells will be difficult. For an acoustic climber, the challenge is the vertical length, 7 kilometers for a 1 Hz tether vibration rate.

However, we do not need acoustic power to move; lowering the climber and motor-sets can generate motor power from the gravitational descent. Anchoring the lower motor-set, gravity can lower the upper motor-set, letting the quarter wavelength spar-tether droop below the bottom of the lower motor-set.

Acoustic climber upper and lower motor-sets with 1 Hz power are separated by 1/4 wavelength, 7000 meters, and cannot extract high levels of acoustic power without that separation. Gravitational energy is still available, so the transfer process is powered by lowering the motor-sets, and letting the connecting spar tether sag downwards. The diagram shows a tilted tether; this presumes that the ground node is offset to the south.

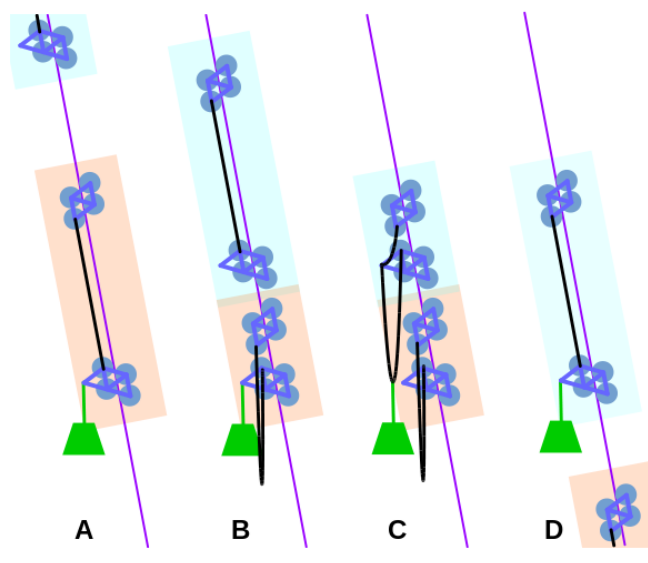

Figure 16: **Transferring payload from a lower to an upper acoustic climber**

The handoff between a "donor" climber below, and an "acceptor" climber above, can be accomplished by lowering both climbers about 10 kilometers, as illustrated in fig. 16. The tether is shown slightly tilted, because the ground node is located south of the equator as proposed in [18], facilitating a preferred slightly-downwards orientation for the climbers. The drooping spar-tether from the upper acceptor climber can reach down past the lower motor-set of of the lower donor climber, and snag the payload, supporting it with its upper motor-set while the lower donor climber descends out of the way. Then the lower motor-set of the upper receiver climber descends, and attaches to the payload.

Fig. 16A. The lower "donor" climber, with payload attached, awaiting transfer to the descending upper "acceptor" climber.

Fig. 16B. The upper motor-set of the lower donor climber descends, "powered" by gravity; now the upper acceptor climber can descend within a few hundred meters vertically.

Fig. 16C. The upper climber lowers its upper motor-set and its spar-tether droops down low enough to attach to the payload. The lower climber is now free to descend; first the lower motor-set, then (after the lower climber straightens out), the entire climber descends, capturing the energy of descent with its motor-sets and broadcasting it downwards as an acoustic transmitter to climbers below.

Fig. 16D. The lower climber descends, and the lower motor-set of the upper acceptor climber descends to re-attach the payload at the bottom rather than the middle of the spar tether.

The transfer requires precision coordination between both climbers, and will occur at specific altitudes on the main space elevator tether. The tether may be enhanced at those altitudes to facilitate transfers.

Note that climber descent generates gravitational energy, which must be transmitted away, or a descending motor-set will rapidly overheat. A 200 kilogram motor-set descending 7 kilometers in a 0.5 gee gravity field produces 7 megajoules of descent energy; if the specific heat of the motor-set materials is 500 J / kg-K, the motor-set would heat 70 Kelvin.

If the entire 2000 kilogram acoustic climber descends 1800 kilometers at 100 meters per second in a 0.5 gee gravity field, the motor-sets produce 1 megawatt of descent power, or 1.8 trillion joules, 900 million joules per kilogram. That would be enough to heat it to 1.8 million degrees if the power was not transmitted acoustically, or radiated as heat, during the 5 hour descent.

Acoustic power recycling is preferable. However, the descent power from the motor-sets can also be radiated as heat energy from long, wide tungsten filaments inside highly polished reflectors. A square meter of filament at 3000 K can radiate 3.7 MW isotropically; confined to a mirror, and with low radiation efficiency, a megawatt of thermal emission should be possible in a radiator system with modest weight.

The motor-sets will themselves be inefficient, so their waste heat must be pumped with refrigerators and radiated at fairly high temperatures for efficient cooling while consuming limited mass in the climber mass budget.

10 Payload rates

A climber needs many megawatts of power to climb out of the lower gravity well. For maximum throughput, it is desirable to do so quickly, because the lower tether cannot support more than one climber against gravity. The power needed for climb is the gravitational force multipled by climber speed, so higher speeds and lower altitudes (with higher gravity) require more power, as shown in fig. 15.

At the bottom of the tether, at 9.8 m/s^{2}with a taper ratio of 6, the tether is 0.013 kg/m, the static tether stress is 350 kN, and the acoustic impedance is 325 kg/s, so a 150 m/s wave corresponds to acoustic power of 2.0 MW and a dynamic stress of 40 kN.

A 16 tonne climber corresponds to about 160 kN of gravitational weight. 2.5 MW will lift it from the ground node at 16 m/s. Early cargos may include an acoustic transmitter and solar power array at GEO, allowing power transmission from above, but that complication is beyond the scope of this paper.

The tether supports climbers, and its own gravitational weight. An acoustic climber is less massive, so some of the tether force can be re-allocated for acoustic vibration, as shown in figure. 17.

The space elevator tether is tapered, less stiff and strong and massive at the ground attachment, so power transmission from the ground is limited to a few megawatts. Minimum vibration tension cannot be less than zero, the **yellow** area in the diagram.

Maximum vibration cannot exceed the derated strength of the tether. Climbers will extract acoustic power to drive climb motors, while leaving some power to propagate upwards to climbers above.

Farther above ground, gravity lessens, and climbers need much less power to climb at high speed, while putting less total tension on the tether, allowing the tension below and above the climber to be increased, and the amount of transmitted power (proportional to vibration force squared) to be increased as well.

At higher altitudes, as the tether cross section increases, the acoustic impedance increases proportionally, which increases the tether vibration force per watt.

The force and power transmitted upwards can be adjusted to match climber position and power needs. However, power may need many minutes to propagate to the climber. From the ground node to a climber at 2240 kilometers altitude, at 28 km/s, power needs 80 seconds to arrive, so the power transmitter must anticipate climber needs.

If a climber fails, or falls off the tether, its acoustic load vanishes, and the power sources at top and bottom should change from power transmission to power absorption. However, the acoustic power may take as much as an hour to drain from the tether, and unexpected resonances may stress the tether to the breaking point. Failure modes will require much further study.

When fully deployed, with a transmitter at GEO, climbers above 1000 km altitudes can ascend at 100 meters per second, making the transit to GEO in a little over four days. The limit on throughput will be the weight of a single climber on the lowest 1000 km of the tether, limiting throughput to one or two 16 tonne climbers per day. Climber mass and speed should be optimized for quick passage through this region. Lighter climbers put less stress on the tether, allowing more of the peak stress to be used for moving acoustic power.

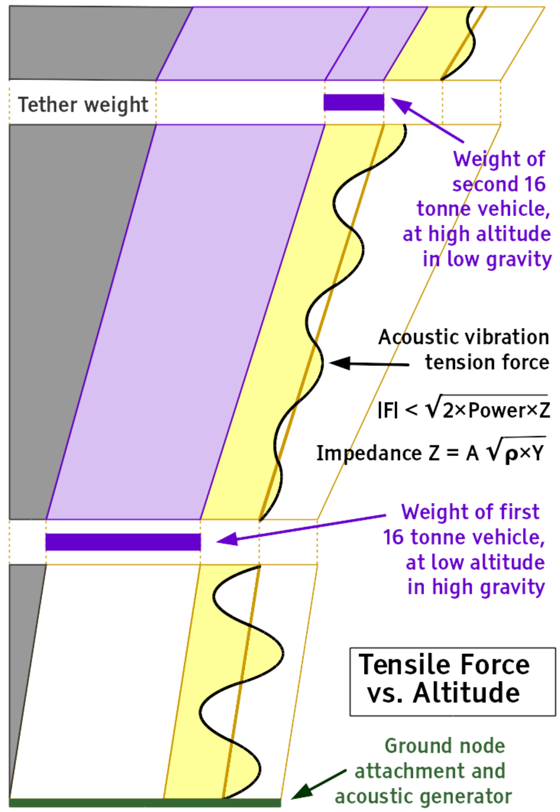

Figure 17: **Tether tension versus altitude**

11 West Elevators for the Launch Loop

Launch Loop version 2.0 will be greatly improved over version 1.0. Among other improvements:

- Drag on the rotor has been replaced with a rotor field slot and aluminum windings in the track that interact with the payload. This increases launch energy conversion efficiency from 40% to better than 98% . The track windings double as a Whipple shield for meteorite and space debris protection. Note that hypothetical conductors lighter-weight than aluminum would reduce resistance losses and waste heat.

- The launch track is moved up to 100 km altitude, producing far less vehicle drag and making large incoming space debris impactors easier to predict and avoid.

- A 1MN Kevlar tension spine down the launch track, connected to a curved incline with a similar tension spine, will permit more than 2MN of thrust to be distributed among launch vehicles. This is horizontal thrust, and permits launch cadences of 240 6 tonne vehicles (5 tonnes payload fraction) per hour. Note that a material with 10 times the strength would permit 10 times the thrust and 10 times the mass throughput,

- West station, where vehicles and launch sleds are loaded onto the track, has been lowered to 50 kilometers altitude.

The increase in maximum vehicle rate requires as many as 300 vehicles to be delivered to and from west station per hour, including launch operations crews, repair parts, and consumables such as coolant ice (turning deflection magnet waste heat into steam, which falls from the station). Acoustic elevators, more

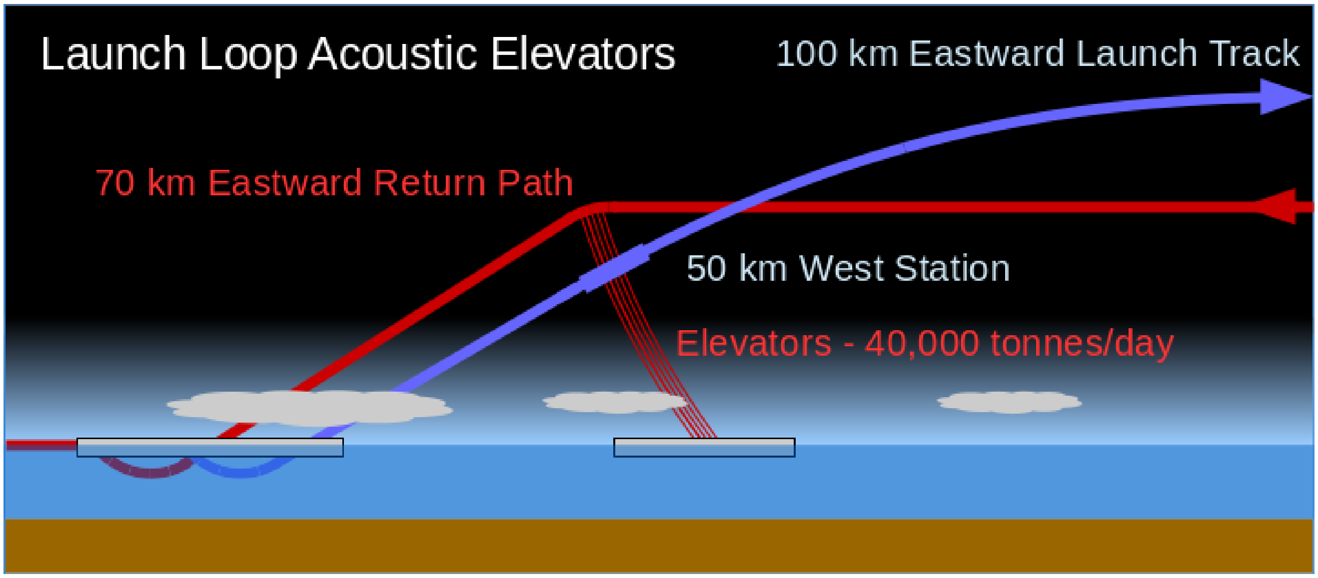

Figure 18: Launch loop elevator system. Dozens of Kevlar elevators carry 7 tonne acoustic climber vehicles from the surface to the West station of the launch loop, where the launch vehicles are tested then propelled up the launch track.

than 50 running in parallel, can deliver 36 thousand tonnes to west station per day. This "minimum" launch loop design can launch five tonne payloads every 15 seconds.

In the distant future, heavier terawatt class launch loops, supplied with 100W/kg space solar power, can launch dozens of 30 tonne intermodal shipping containers per minute, and may need hundreds of elevators.

The west station elevators will be round Kevlar 49 elevator cables, one kilogram per meter, derated by a factor of four from ultimate strength. Ribbon or mesh tethers may accumulate too much ice near the tropopause, though they would be easier to climb. De-icing strategies are needed.

The speed of sound in Kevlar is presumed to be 6 km/s, and the weave of the cable is presumed to be optimized for minimum acoustic hysteresis. With an acoustic frequency of 6 Hz, the wavelength is 1 kilometer, a climber is 250 meters high, and the entire elevator will be 50 wavelengths high.

The hysteresis losses in the Kevlar itself can be less than 0.5%, though an epoxy binder can increase that to 2%. Even if the acoustic losses in properly-designed, modestly-loaded Kevlar are 1% per cycle, 60% of the acoustic power transmitted down from west station will reach the acoustic climber on the ground (where speed is limited by air drag) and 72% will reach the climber as it climbs above the 17 km equatorial stratopause.

Strong fibers will be used in kilotonne quantities in the launch loop; if the price of stronger carbon fibers becomes competitive with Kevlar on a per-MN basis, launch loop performance increases proportionally.

Climb vehicle cross sections and "wing loadings" will resemble a Cessna 172-class airplane. Climb vehicles will weigh 7 tonnes; 5 tonnes for the payload, 1 tonne for the launch sled, and 1 tonne for the acoustic climber.

Velocity below 20 km altitude will be drag limited, but most of the climb will be limited by wheel speed. Like the space elevator acoustic climbers described in the paper, magtread wheels can permit higher velocities.

Climb time from surface to west station will be around 10 minutes. It may be possible to stack more than one climber on an elevator cable, but for safety and logistics reasons, we will limit throughput to one climber at a time.

Unlike the more sophisticated staged climber system described above and elsewhere [18], these climbers will go straight to the west station destination, dispense their payloads, then climb an additional 300 meters above west station, to join as many as 10 other climbers waiting for a trip back to the surface.

One elevator cable should be able to average five upbound deliveries per hour, so 60 elevators working in parallel should be able to handle 300 vehicles per hour.

The cables continue from west station on the eastbound rotor to a large support deflector on the westbound (return) rotor, at 60 kilometers altitude. This support deflector will have power extraction generators that provide more than 500 megawatts to the elevator acoustic transmitters

If the elevator climber breaks, there's plenty more where that came from. The vehicles themselves will have stubby wings, which they can use for emergency reentry during or after a launch; the wings (plus lots of telemetry and remote computer control) will help the vehicles get out of the way of a crashing elevator cable. The cables should be spaced far enough apart to limit fratricide.

Lightning is an unsolved problem. The acoustic elevator cables pass through the topical tropopause, and thunderstorms can gather just below. The west station elevator cables should be grouped, and surrounded by an additional ring of high conductivity cables and lightning rods to prevent lightning from hitting the high resistance Kevlar elevator cables.

The launch loop can be constructed with existing engineering materials; materials improvements and other inventive solutions pursued by the space elevator community will make launch loops cheaper and more productive. Even if space elevators prove impractical, the work of the space elevator community applies directly to launch loops and other launch alternatives, so our collaborative cross-fertilization efforts will advance our common goals and ensure future successes.

12 Conclusion

Space elevator tethers must be extremely strong and stiff, allowing them to carry huge amounts of power as longitudinal acoustic vibrations. The vibration can be transmitted from power sources on the ground or at GEO node, sent through the tether with high efficiency, and converted to climber velocity and thrust with receivers on each climber.

Acoustic power transmission is a preliminary and incomplete idea. It will need much more development to be practical, requiring a clever and durable design for an efficient acoustic receiver that produces mechanical thrust without too much expensive-to-radiate heat.

However, even a low efficiency system will be less expensive and weigh far less than solar panels supporting themselves, their structure, and their electrical wiring against gravity.

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